Competitive Cheap Talk

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Abstract

This paper studies a competitive cheap talk model with two senders. Each sender, who is responsible for a single project, only observes the return of his own project. Exactly one project will be implemented. Both senders share some common interests with the receiver, but at the same time have own project biases. Under simultaneous communication, all equilibria are shown to be partition equilibrium, and the partitions of two agents' are intimately related: the interior partition points of two agents has an alternating structure. In the most informative equilibrium, the agent with a smaller bias has weakly more partitions. Simultaneous communication, sequential communication and simple delegation are essentially all outcome equivalent, as they always lead to the same most informative equilibrium.

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1 Introduction

Decision makers often seek advice from multiple experts. For instance, consider an economics department trying to hire a junior faculty member. The two targeted fields are, say micro theory and macro. Due to budget constraint exactly one position will be filled. In each field a single candidate is identified. The theory group of the department observes the quality of the theory candidate but not that of the macro candidate. Similarly, the macro group observes the quality of the macro candidate but not that of the theory candidate. The department chair, say a labor economist, does not observe the quality of either candidate. The chair prefers to hiring the candidate of higher quality. For each group, though they also prefer the higher quality candidate being hired, they have own-field biases: if the candidate of a group is hired that group derives a positive private benefit.

The above example has several distinguishing features. (i) A decision maker (DM) consults two experts regarding two alternative options (projects). (ii) The experts' interests are largely aligned with the DM's, but each expert has his own-project bias. (iii) Two experts only observe the return

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of his own project. (iv) The DM's action is binary (which project to adopt) and exactly one project will be adopted. Thus in some sense two experts are competing with each other in having their own projects adopted. The purpose of this paper is to study communication or information transmission in the above setting, with communication being modeled as cheap talk (Crawford and Sobel, 1982, CS hereafter). The novelty of the paper is that we introduce an aspect of competition explicitly into cheap talk models with multiple senders: each sender has an extra incentive to have his own project implemented at the expense of the other sender.

Real world situations of competitive cheap talk, which share the above features, abound. For instance, consider a CEO of a firm deciding on launching one of two alternative new products (projects). The CEO consults two managers, who each are responsible for one of the two products, regarding the profitability of each product. Each manager only knows the profitability of his own product and has an extra incentive to have his own product launched. Alternatively, consider the President weighing between two alternative policies to address the environmental issues. The President consults two experts who each are responsible for investigating the effectiveness of one policy. Each expert only observes the effectiveness of his own policy but have an extra incentive to have his own policy adopted.

Specifically, there are two symmetric projects and the return of each project is uniformly distributed. The DM's payoff is just the return of the adopted project. Each agent's payoff has two components. The first component is the return of the adopted project. This component implies that the two experts' and the DM's interests are largely aligned: all prefer the project with a higher return being implemented. The second component is private benefit: an agent reaps this payoff if and only if his own project is adopted. We call this component own-project bias, which creates conflicts of interests: each agent prefers having his own project implemented. Given that exactly one project will be implemented, two agents' own-project biases create competition between two agents. We mainly focus on the case of asymmetric agents, with agent 1 having a smaller bias.

We first study the situation in which two agents send messages simultaneously. As in standard cheap models, all equilibria are shown to be partition equilibrium in which each agent only indicates to which interval the return of his own project belongs. Any equilibrium must be asymmetric, meaning that agents have different partitions. The combined messages of two agents can be ranked according to the posterior induced (low messages versus high messages).

The first main result of the paper is that the equilibrium information transmissions of two agents are intimately related. In particular, in equilibrium two agents' messages must exhibit an alternating ranking structure: for any message belonging to one agent the two messages of adjacent rankings must belong to the othe agent. Correspondingly, two agents' interior partition points also have an alternating feature: one agent's partition point must be neighbored by two partition points of the other agent. As a result, in equilibrium two agents either have the same number of partitions, or the numbers of partitions differ by one. This implies that the amount of (meaningful) information transmitted by two agents cannot be too far apart. Moreover, if one agent's bias decreases, then both agents will transmit more information in the most informative equilibrium. Thus in some sense two agents' information transmissions are strategic complements. The underlying reason for these features is as follows. The DM's problem is to select the better project to implement. Thus it is the comparison of two projects' returns that matters. If one agent transmits much more information than the other agent does, then some information transmitted by the first agent will be wasted as it cannot improve the DM's decision making. When one agent's bias decreases, this agent will naturally transmit more information, and this also allows the other agent to transmit more (meaningful) information.

The lowest (highest) overall message has the feature that if an agent sends that message then that agent's project will not (will) be implemented for sure regardless of the other agent's message. We call the lowest and highest overall messages as the give-up option and the sure option respectively. We call an equilibrium with agent i having the give-up option an A-i-S equilibrium. In total there are four types of equilibria: A-1-S equilibria with odd or even (total) number of partitions, and A-2-S equilibria with odd or even number of partitions. We focus on the most informative equilibrium, which maximizes the DM's expected payoff, and ask the following question: whether in the most informative equilibrium agent 1, who has a smaller bias, should always have the give-up option? Compared to the most informative A-2-S equilibrium, the most informative A-1-S equilibrium either has one more partition or has the same number of partitions. This is because in A-1-S (A-2-S) equilibrium, agent 1's (agent 2's) bias, which is smaller (bigger), enters into the incremental step sizes more often. If an A-1-S equilibrium and an A-2-S have the same odd number of partitions, then the A-1-S equilibrium is more informative, as it leads to overall more even partitions. However, if an A-1-S equilibrium and an A-2-S have the same even number of partitions, then the A-2-S equilibrium is more informative. The general conclusion is that, in the most informative equilibrium it is not always the case that the agent with a smaller bias has the give-up option. However, it is more likely that the agent with a smaller bias has the give-up option in the most informative equilibrium.

Similarly, in the most informative equilibrium, it is not always the case, but it is more likely, that the agent with a smaller bias has the sure option. Moreover, relative to the agent with a bigger bias, in the most informative equilibrium the agent with a smaller bias has weakly more messages. Finally, in the most informative equilibrium, while it is possible for the agent with a smaller bias to have the give-up option and sure option at the same time, it is impossible for the agent with a bigger bias to have both options at the same time. All these properties indicate that the agent with a smaller bias is chosen to be trustworthy more often in equilibrium.

We also ask the following comparative statics question: fix the combined bias of two agents, will the DM be better off or worse off when two agents' biases become relatively more unequal? As two agents' biases become more unequal, while the number of partitions of the most informative A-1-S equilibrium increases, that of the most informative A-2-S equilibrium decreases. However, the overall most informative equilibrium could become more informative or less informative. The general conclusion is that the DM's payoff depends both on the combined bias and the distribution of biases between two agents. Interestingly, the DM's payoff in the most informative equilibrium could increase even when the combined bias increases.

We then study sequential communication (talk) in which one agent publicly sends message first. Since the second agent is able to condition his message on the first agent's message, the second agent can at most have two messages, which essentially indicates whether the return of his own project is higher than that of the first agent's project. Interestingly, the set of equilibria under simultaneous communication (talk) and that under sequential communication are equivalent or lead to the same outcomes. This is a quite surprising result, as in typical cheap talk models with multiple senders sequential communication and simultaneous communication usually lead to different outcomes. The underlying reason is again due to the fact that only the comparison of two projects matters. Roughly speaking, even under simultaneous talk, when the marginal type of one agent decides which message to send, he has already implicitly conditioned on that the other agent's message has adjacent rankings. This implies that, under sequential talk, the second agent's ability to directly condition his message on the first agent's message does not matter.

We then consider simple delegation, under which the DM delegates the decision rights to one of the agents. It turns out that simple delegation and sequential talk are essentially equivalent, in the sense that they lead to the same most informative equilibrium. This is because under sequential talk the DM's decision always follows the second agent's message; it is as if the second agent have the decision rights. Combining with previous results, we conclude that simultaneous talk, sequential talk, and simple delegation all lead to the same most informative equilibrium. This is quite surprising as in other cheap talk models delegation will lead to outcomes different from those under simultaneous talk or sequential talk.

Comparing two agents' payoffs, under simultaneous communication it turns out that in any equilibrium the agent who has the sure option is always better off than the other agent. Translated into other settings, the agent who talks the second under sequential communication and the agent who has the decision rights under simple delegation always gets a higher expected payoff.

Finally, we study the case of more than two agents. Our focus is on symmetric agents of the same bias. In the most informative symmetric equilibrium, we show that as the number of agents increases each agent transmits more information. This implies that more intense competition among agents leads to more information transmission. Intuitively, with more agents it is more likely that there is at least one agent whose project has a higher return. This means that the cost of sending a higher message increases for each agent, which reduces each agent's incentive to exaggerate the return of his own project.

This paper is related to the growing literature on cheap talk with multiple senders. For some models (Gilligan and Krehbiel, 1989; Epstein 1998; Krishna and Morgan, 2001a, 2001b; Li, 2010), the state space is one dimensional and both senders perfectly observe the same realized state. In Austen-Smith (1993), senders receive correlated (conditionally independent) signals regarding the state. The main difference between those models and ours is that there is no explicit competition between senders in those models. In Krishna and Morgan (2001a), for instance, the receiver's action space is continuous, and two senders try to pull the receiver's action either in different directions (opposing biases) or in the same direction but to different degrees (like biases). In some sense, two senders are competing with each other in influencing the receiver's action to their own favor. But the competition is implicit in that the receiver can combine the information transmitted by both senders and fine tune his action continuously, as his action space is continuous. In our model, the state space is two dimensional (the returns of two projects), and the receiver's action is binary since exactly one project will be implemented. Thus competition is explicit in the sense that only one sender's project will be implemented. Another difference is that in our model two senders observe non-overlapping private information (each only observes the return of his own project).

Hori (2006) and Yang and McGee (2013) study cheap talk models in which two senders have partial and non-overlapping private information, and the receiver's action space is one dimensional but continuous. Alonso et al. (2008), Rantakari (2008), and Yang (2013) study strategic communication with senders having non-overlapping private information, but the receiver's action space is multi-dimensional and continuous (each decision needs to be made for each sender's division). In those models, the need to communicate results from the need to coordinate decisions across different senders. Again, in those models, although each sender tries to pull the receiver's decision(s) to his own favor, there is no explicit competition between senders. Battaglini (2002) and Ambrus and Takahashi (2008) study multidimensional cheap talk models with multiple senders.[1] In both models, each sender observes the realized states in all dimensions and the decision is a two-dimensional vector. In this setting, full information revelation can be typically achieved in equilibrium.

In a two-stage auction setting, Quint and Hendricks (2013) model the first stage indicative bidding as a cheap talk game. The two bidders who send the highest messages will be selected by the seller (receiver) to advance to the second stage of auction. In some sense, bidders in the first stage, by cheap talking, are competing with each other for the two spots in the second stage. This aspect is pretty much related to our paper. While their model focuses on the setting of indicative bidding, our model applies to more general situations of competitive cheap talk. The most important difference is that in their model there is only pure conflict of interests among the bidders (senders), while in our model senders have common interests as they care about the quality of the adopted project. Finally, bidders are symmetric in their model as they have the same entry cost, and they focus on the symmetric equilibrium. In our model senders have different biases and we focus on asymmetric equilibrium.

This paper is also related to "comparative" cheap talk (Chakraboty and Harbaugh, 2007, 2010; Che et al., 2013). In those models, a single expert observes the realized returns of multiple projects, and makes recommendation to the receiver, who then makes decision about which project to implement. Under certain conditions, Chakraboty and Harbaugh (2007, 2010) show that some information can be credibly transmitted by the expert by making comparative statements. Focusing on asymmetric projects, Che et al. (2013) find that pandering is possible: the expert sometimes might recommend a "conditionally better-looking" project whose realized return is lower than that of the other project. Our paper is related to those papers in that the receiver's action is binary (which project to implement).[2] The main difference is that in our model there are multiple experts, and each expert is responsible for a single project. This creates an aspect of competition in our model, which is absent in their models.

The rest of the paper is organized as follows. Section 2 sets up the model and offers some preliminary analysis. Section 3 studies a benchmark case of simultaneous communication with symmetric agents. In Section 4 we study simultaneous communication with asymmetric agents. To characterize asymmetric equilibrium, we introduce and study an equivalent equilibrium which we call quasi-symmetric equilibrium. Section 5 studies sequential communication and simple delegation, and the case of more than two agents are investigated in Section 6. Section 7 offers conclusions and discussions.

[1] Hagenbach and Koessler (2010) and Galeotti et al. (2013) study strategic communication in networks. Each agent is sender and receiver at the same time. The need to coordinate actions among agents gives rise to the need to communicate. In both models, agents' private information is binary.

[2] Jindapon and Oyarzun (2013) also study a one-sender cheap talk model in which the receiver takes a binary action as to whether to accept a good recommended by the sender. The sender has two possible types, honest or biased, and his type is unobservable to the receiver.

2 Model and Preliminary Analysis

Consider a principal or a decision maker (DM) who is facing the choice between two alternative projects. The return of project i, $i = 1, 2$, is θ_i, which is uniformly distributed on $[0, 1]$. We assume that θ_1 and θ_2 are independent from each other. There are two agents, with each agent i being responsible for investigating project i. The realization of θ_i is only observed by agent i.[3] The DM has to adopt exactly one project. Adopting both projects is not feasible, which could be possibly due to some budget constraint. This implies that the two projects are competing projects. Adopting neither project is not an option either.[4]

In the basic model we consider the case of simultaneous communication, with two agents simultaneously sending messages to the DM. Denote agent i's message as m_i. After hearing messages m_1 and m_2, the DM decides which project to adopt. Let $d \in \{1, 2\}$ be the DM's decision, with $d = i$ indicating that project i is adopted.

Given the project choice d, the DM's payoff is $U_P(d) = \theta_d$. Agent i's payoff is given by

$$U_i(d) = \begin{cases} \theta_d & \text{if } d \neq i \\ \theta_d + b_i & \text{if } d = i \end{cases} .$$

The parameter $b_i \in (0, 1)$ represents agent i's own project bias. That is, agent i derives private benefit b_i if and only if project i is adopted. Both b_1 and b_2 are common knowledge. Observing the payoff functions, we see that there is a common interest among the DM and two agents: all of them care about the return of the adopted project and want to choose the project that has a higher return. The conflict of interest is reflected in the own project biases b_1 and b_2: each agent has a bias to have his own project to be adopted. If b_i were 0 for both agents, then agents' interests are perfectly aligned with the DM's. Note that b_1 and b_2 could be different, or two agents could have different biases. Without loss of generality, we assume that $0 < b_1 \leq b_2$. That is, agent 1 has a smaller bias. To ensure that some information could possibly be transmitted in equilibrium, we further assume that $b_1 \leq b_2 < 1/2$.[5] All players are expected utility maximizers.

One rationale for such payoff structure is as follows. The profit of the firm, owned by the DM, is θ_d. Each agent is paid a fraction of the firm's profit, say $\alpha\theta_d$, and each agent derives private benefit B_i if his own project is chosen. The DM's net payoff is $(1 - 2\alpha)\theta_d$, which is proportional to θ_d. Defining $b_i = B_i/\alpha$. Then agent i's utility function is proportional to $U_i(d)$. The private benefits of agents could be due to many reasons. In the example of department hiring, the private benefit could be that hiring a candidate of the same field might lead to future cooperation with existing group members. In the example of the firm, the manager whose project is chosen is very likely to be the one who will carry out the project, the action of which usually brings private benefit. In the example of government policy, the future career of the expert whose policy is chosen might be

[3] This feature that different agents observe different information is understudied in the cheap talk literature. It is reasonable due to specialization in the modern world: in organizations such as firms and governments, different divisions (groups) specialize in different functional areas.

[4] Later on we will discuss what will happen if there is a third option of implementing neither project.

[5] In particular, this condition implies that if one agent babbles then it is possible for the other agent to transmit some information.

boosted by the very fact that his policy is chosen.

Under simultaneous communication, a strategy for agent i specifies a message m_i for each θ_i, which is denoted as the communication rule $\mu_i(m_i|\theta_i)$. A strategy for the DM specifies an action d for each message pair (m_1, m_2), which is denoted as decision rule $d(m_1, m_2)$. Let the belief function $g(\theta_1, \theta_2|m_1, m_2)$ be the DM's posterior beliefs on θ_1 and θ_2 after hearing messages m_1 and m_2. Since θ_1 and θ_2 are independent and agent i observes only θ_i, the belief function can be decomposed into distinct belief functions $g_1(\theta_1|m_1)$ and $g_2(\theta_2|m_2)$.

Our solution concept is Perfect Bayesian Equilibrium (PBE), which requires:

(i) Given the DM's decision rule $d(m_1, m_2)$ and agent j's communication rule $\mu_j(m_j|\theta_j)$, for each i, agent i's communication rule $\mu_i(m_i|\theta_i)$ is optimal.

(ii) The DM's decision rule $d(m_1, m_2)$ is optimal given beliefs $g_1(\theta_1|m_1)$ and $g_2(\theta_2|m_2)$.

(iii) The belief functions $g_i(\theta_i|m_i)$ are derived from the agents' communication rules $\mu_i(m_i|\theta_i)$ according to Bayes rule whenever possible.

Given two agents' strategies, the DM's optimal decision is just to implement the project that has a higher expected return. That is, the optimal decision can be written as

$$d(m_1, m_2) = \begin{cases} i & \text{if } E[\theta_i|m_i] > E[\theta_j|m_j] \\ j & \text{if } E[\theta_i|m_i] < E[\theta_j|m_j] \\ i \text{ or } j & \text{if } E[\theta_i|m_i] = E[\theta_j|m_j] \end{cases} . \tag{1}$$

And the DM's expected (interim) payoff given m_1 and m_2 is given by $E[U_p(m_1, m_2)] = \max\{E[\theta_1|m_1], E[\theta_2|m_2]\}$.

As in CS and ADM, all PBE are interval equilibria. Specifically, the state space $[0, 1]$ is partitioned into intervals and agent i only reveals to which interval θ_i belongs.

Proposition 1 *All PBE in the simultaneous communication game must be interval equilibrium.*

Proof. Since the two agents' situations are symmetric, we only need to provide a proof for agent 1. Let $\mu_2(\cdot)$ be any communication rule for agent 2. Suppose the realized return of project 1 is θ_1 and agent 1 induces a posterior belief v_1 of θ_1. Given the DM's optimal decision (1), agent 1's expected utility can be written as

$$E_{\theta_2}[U_1|\theta_1, v_1] = \Pr(E[\theta_2|\mu_2(\cdot)] \le v_1)(\theta_1 + b_1) + \Pr(E[\theta_2|\mu_2(\cdot)] > v_1)E[\theta_2|E[\theta_2|\mu_2(\cdot)] > v_1].$$

From the above expression, because $\Pr(E[\theta_2|\mu_2(\cdot)] \le v_1)$ is increasing in v_1, it can be verified that $\frac{\partial^2}{\partial\theta_1\partial v_1}E_{\theta_2}[U_1|\theta_1, v_1] > 0$. This means that for any two different posterior of θ_1, say $\underline{v}_1 < \overline{v}_1$, there is at most one type of agent 1 who is indifferent between $\underline{v}_1$ and $\overline{v}_1$. Now suppose there is a PBE which is not an interval equilibrium. In particular, there are two states $\theta_1^1 < \theta_1^2$ such that $E_{\theta_2}[U_1|\theta_1^1, \overline{v}_1] \ge E_{\theta_2}[U_1|\theta_1^1, \underline{v}_1]$ and $E_{\theta_2}[U_1|\theta_1^2, \overline{v}_1] < E_{\theta_2}[U_1|\theta_1^2, \underline{v}_1]$. It follows that $E_{\theta_2}[U_1|\theta_1^2, \overline{v}_1] - E_{\theta_2}[U_1|\theta_1^2, \underline{v}_1] < E_{\theta_2}[U_1|\theta_1^1, \overline{v}_1] - E_{\theta_2}[U_1|\theta_1^1, \underline{v}_1]$, which violates the fact that $\frac{\partial^2}{\partial\theta_1\partial v_1}E_{\theta_2}[U_1|\theta_1, v_1] > 0$. Therefore, any PBE must be interval equilibrium. ■

Essentially, due to own project bias each agent tries to overstate the return of his own project to some extent. The benefit of overstating, say by agent 1, is that agent 1's project will more likely be implemented and thus agent 1 is more likely to reap the private benefit. On the other hand, there is a cost of overstating: overstating by agent 1 reduces the probability that agent 2's project will be implemented, which might have a higher return. Consider two different types of agent 1 reporting as the same (higher) type. Compared to the lower type, the overstating of the higher type involves a smaller cost. This is simply because, with the higher type project 1 has a higher return and is more likely than project 2 to be the better project. Therefore, a higher type of agent 1 will try to induce a higher posterior, which implies that all PBE must be interval equilibrium.

3 Simultaneous Communication with Symmetric Agents

Before studying the more general case in which two agents have different biases, we first consider the case that two agents are symmetric, which serves as a benchmark. More specifically, suppose two agents have the same bias: $b_1 = b_2 = b$. We are interested in symmetric equilibrium, in which both agents adopt the same strategy and the DM treats two agents equally.[6] Recall that Proposition 1 establishes that all PBE must be of interval or partition form. Let $\{a_n\}$ be the partition points and N be the number of partitions. Each agent i sends message m^n if $\theta_i \in [a_{n-1}, a_n]$, with $a_0 = 0$ and $a_N = 1$. The DM's optimal decision rule is easy to describe: implement the project that has the higher expected payoff; if there is a tie then implement two projects with equal probability. Given agents' information transmission strategy, $E[\theta_i|m_i^n] = (a_{n-1} + a_n)/2$. We say that a message m^n is a higher message if n is larger. In general, the DM will adopt the project of the agent whose message is higher.

To characterize the equilibrium partition points $\{a_n\}$, consider agent 1. If $\theta_1 = a_n$, $1 < n < N$, agent 1 should be indifferent between sending message m^n and message m^{n+1}. More explicitly, this indifference condition can be written as:

$$\frac{1}{2}(a_{n+1} - a_n)\frac{a_{n+1} + a_n}{2} + \frac{1}{2}(a_n - a_{n-1})\frac{a_n + a_{n-1}}{2} = [\frac{1}{2}(a_{n+1} - a_n) + \frac{1}{2}(a_n - a_{n-1})](a_n + b). \quad (2)$$

To understand equation (2), note that, for agent 1, sending message m^n or message m^{n+1} matters for the outcome (which project is implemented) only when agent 2's message is either m^n or m^{n+1}. If agent 2's message is higher (lower) than m^{n+1} (m^n), project 2 (project 1) will be implemented regardless of agent 1's message. By sending message m^{n+1}, agent 1 increases the probability that project 1 will be implemented when agent 2's message is either m^{n+1} or m^n. The gain in expected payoff of agent 1 is captured by the RHS of (2). In the mean time, the probability that project 2 will be implemented is decreased. The loss in expected payoff of agent 1 is captured by the LHS of (2).

[6]Asymmetric equilibria exist for symmetric agents, which are qualitatively similar to the equilibria we derived for asymmetric agents.

Equation (2) can also be written as

$$\frac{1}{2}(a_{n+1}-a_n)[\frac{a_{n+1}+a_n}{2}-(a_n+b)] = \frac{1}{2}(a_n-a_{n-1})[(a_n+b)-\frac{a_n+a_{n-1}}{2}]. \tag{3}$$

Equation (3) has a clear interpretation. The RHS is the expected gain of sending the higher message (when the other agent sends the lower message), and the LHS is the expected loss of sending the higher message (when the other agents sends the higher message). Equation (2) can be further simplified as:

$$(a_{n+1}-a_n)-(a_n-a_{n-1}) = 2b. \tag{4}$$

Equation (4) indicates that the partition sizes are increasing in the direction of b, with the incremental step size being $2b$. This feature is very similar to the partition equilibrium in the one-sender model of CS (the uniform-quadratic setting), where the incremental step size is $4b$. The similarity in equilibrium features is surprising, given that our model is quite different from the CS model. In particular, in CS there is only one sender, both the DM and the agent have quadratic loss utility functions, and the DM's action is continuous, while in our model there are two senders, both the DM and the agents have linear utility functions, and the DM's action is binary. In the current model, the partition size increases because the cost of overstating has to be endogenously created. If the partition sizes were equal, then the marginal type would strictly prefer to report the higher message. By increasing the partition size for the higher message, it increases the cost of reporting the higher message for the marginal type: the other agent is more likely to report the higher message and conditional on that the other agent's project has a higher return.

Combining with the boundary conditions, the difference equation (4) can be solved as $a_n = \frac{n}{N} - n(N-n)b$. The constraint that the total length of all partitions is less than 1 leads to the inequality that $bN(N-1) < 1$, which gives rise to the upper bound of N, $\overline{N}$: $\overline{N} = \langle \frac{1}{2}+\frac{1}{2}(1+\frac{4}{b})^{1/2}\rangle$.

Just like standard cheap talk models, given $\overline{N} > 1$, for any integer N such that $1 \leq N < \overline{N}$ there is a symmetric equilibrium. The DM's expected payoff in the N-partition equilibrium, $E(U_p)$ can be calculated as:

$$\begin{aligned} E(U_p) &= \sum_{n=1}^{N}[(a_n-a_{n-1})^2 + 2(a_n-a_{n-1})a_{n-1}]\frac{a_n+a_{n-1}}{2} \\ &= \frac{2}{3}-\frac{1}{6N^2}-\frac{b^2(N^2-1)}{6}. \end{aligned} \tag{5}$$

From the expression of (5), we can see that $E(U_p)$ is decreasing in b, and $E(U_p)$ is increasing in N for $N < \overline{N}$.[7]

The first term of $E(U_p)$ in (5), 2/3, is the expectation of the first order statistic of two random variables that are uniformly distributed on $[0, 1]$. That is, 2/3 is the expected payoff the DM can get if both agents fully reveal their private information. The last two terms reflect the payoff loss

[7]To see the last property, note that

$$E(U_p(N)) - E(U_p(N-1)) \sim 1 - b^2N^2(N-1)^2 > 0,$$

where the last inequality follows that $bN(N-1) < 1$.

or inefficiency when two agents do not fully reveal information. In equilibrium, inefficiency does not arise when two agents send different messages since in such a case the project with a higher payoff is always implemented. Inefficiency arises only when two agents send the same message. In such a case the principal randomly adopts one project, and the adopted project could have a lower payoff. When the number of partition increases, the probability that two agents send the same message decreases, which decreases the probability that the wrong project is implemented. If the bias b decreases but the number of partitions remain the same, the partitions will be of more even size. The overall probability that two agents send the same messages will decrease,[8] which reduces the probability that the wrong project is adopted.

From ex ante point of view, each agent's expected payoff is just $E(U_p) + b/2$, since each agent's project will be implemented with probability $1/2$. Thus the most informative equilibrium with $\overline{N}$ partitions is also the Pareto dominant equilibrium.

4 Simultaneous Communication with Asymmetric Agents

Now we go back to the general setting with asymmetric agents. Throughout this section we assume that $b_1 < b_2$. The following lemma shows that there is no symmetric informative equilibrium. Note that symmetric babbling equilibrium always exists: two agents babble and the DM ignores the messages and randomly selects one project to implement with equal probability.

Lemma 1 *There is no symmetric informative equilibrium in which two agents have the same partitions and the DM implements both projects with equal probability if two agents send the same message.*

Proof. Suppose to the contrary, there is a symmetric informative equilibrium, with both agents having the same $N \geq 2$ partitions, and the DM implements two projects with equal probability if two agents send the same message. Let $\{a_n\}$ be the equilibrium partition points. By previous analysis for symmetric agents, the indifference conditions that characterize $\{a_n\}$, (4), should be satisfied for both agents. That is, we must have

$$\begin{aligned}
(a_{n+1} - a_n) - (a_n - a_{n-1}) &= 2b_1, \\
\text{and } (a_{n+1} - a_n) - (a_n - a_{n-1}) &= 2b_2.
\end{aligned}$$

But since $b_1 < b_2$, the above two equations cannot be satisfied at the same time. Therefore, symmetric informative equilibrium does not exist. ■

4.1 Asymmetric equilibria

After ruling out symmetric equilibria, it is natural to start with *asymmetric equilibria*, in which two agents have different partitions. Let N_1 and N_2 be the numbers of partitions, and

[8] In a two-partition example, let the partition point be $a_1 \in (0, 1/2)$. The overall probability of tying is $(1 - a_1)^2 + a_1^2$, which is decreasing in a_1 (when two partitions become more even).

$a_1 = (a_{1,0}, a_{1,1}, ..., a_{1,N_1})$ and $a_2 = (a_{2,0}, a_{2,1}, ..., a_{2,N_2})$ be the partition points, for agent 1 and agent 2 respectively. Agent i sends message m_i^n if $\theta_i \in [a_{i,n-1}, a_{i,n}]$. Since two agents' partitions are different, given any pair of messages (m_1, m_2), generically the DM's induced posteriors about two projects will be different (ties are non-generic). Thus the DM generically will only play pure strategies regarding which project to implement. Put it differently, for all possible messages (combining agent 1's and agent 2's) the DM's induced posteriors can be strictly ranked. To ensure that each message is meaningful or outcome relevant, we offer the following definition.

Definition 1 *Two messages of agent i are said to be outcome equivalent if, regardless of the message sent by agent j, sending either of the two messages always lead to the same outcome as to which project is implemented. A set of messages of agent i is said to be irreducible if any pair of messages in the set are not outcome equivalent.*

We will mainly focus on the sets of messages (partitions) that are irreducible, since adding additional outcome equivalent messages will not affect the outcome (unless introducing reducible messages makes the analysis easier). Note that in the equilibria we derived earlier for symmetric agents, the equilibrium sets of messages or (partitions) are irreducible, as sending different messages for each agent will lead to different outcomes with some positive probability.

Recall that for all possible messages associated with an equilibrium the DM's induced posteriors can be strictly ranked. A particular ranking structure is described in the following definition.

Definition 2 *A set of messages is said to have an alternating ranking structure between two agents if (i) the messages having the highest, the 3rd highest, the 5th highest, and so on, posteriors belong to agent i, and (ii) the messages having the 2nd highest, the 4th highest, the 6th highest, and so on, posteriors belong to agent j.*

The following lemma shows the relationship between irreducible sets of messages and the alternating ranking structure.

Lemma 2 *If a set of messages is irreducible, then (i) it must exhibit an alternating ranking structure, (ii) the total number of messages must fall into one of the following three cases: $N_1 = N_2$, $N_1 = N_2 + 1$, and $N_2 = N_1 + 1$.*

Proof. To show part (i), suppose two messages of consecutive overall rankings belong to the same agent, say agent 1. These two messages are outcome equivalent, because which one of the two messages is sent by agent 1 does not affect which project will be implemented: project 2 (1) is implemented if agent 2 sends a higher (lower) message. Therefore, an irreducible set of messages must exhibit an alternating ranking structure. Part (ii) immediately follows part (i), since an alternating ranking structure implies that the total numbers of messages for two agents are either the same ($N_1 = N_2$), or the total number of messages of one agent is that of the other agent plus one (either $N_1 = N_2 + 1$ or $N_2 = N_1 + 1$). ■

Lemma 2 implies that the amount of meaningful information transmitted by two agents are intimately related and cannot be too far apart. To understand the intuition, note that the DM's problem is to select the better project to implement, or it is the comparison of two projects' returns that matters. It means that, to improve the DM's decision, both agents have to transmit more information; and the DM's decision will not improve if only one agent transmits more information. To see this, consider an extreme case in which agent 1 has no bias ($b_1 = 0$) and agent 2 has a very large bias ($b_2 > 1$). In this case, agent 1 will fully reveal his information and agent 2 will reveal no information (babble). Note that although agent 1 fully reveals his information, given that agent 2 reveals no information, his information cannot be fully utilized by the DM in decision making. Actually, the amount of information of agent 1 that can be utilized in decision making is at most of two partitions: whether θ_1 is below $1/2$ (the unconditional mean of θ_2), above $1/2$. If agent 2 reveals more information (say has N partitions), then the meaningful amount of information that can be transmitted by agent 2 increases as well (has $N + 1$ partitions).

In equilibrium, either agent 1 or agent 2 has the lowest overall message (with the lowest induced posterior). Note that if an agent sends the lowest overall message, then that agent's project will not be implemented (the other agent's project will be implemented) for sure regardless of the messages sent by the other agent. That is, an agent sends the lowest overall message means that he "gives up" on his own project. For that reason, we call the lowest overall message the "give-up option." Correspondingly, if an agent sends the highest overall message, then that agent's project will be implemented for sure. We call the highest overall message the "sure option." Note that if the total number of partitions is odd, then the same agent has both the give-up option and the sure option. But if the total number of partitions is even, then one agent has the give-up option and the other agent has the sure option.

Given the alternating ranking structure, we can classifiy asymmetric equilibria by which agent has the give-up option or the lowest overall message. We call equilibria in which agent i has the give-up option as *agent-i-sacrificing (A-i-S) equilibria*. Note that these two types of equilibria are qualitatively similar (just switch the roles of two agents or b_1 and b_2). Alternatively, we can also classify equilibria by which agent has the sure option. But it turns out that equilibrium features depend more on which agent has the give-up option. For this reason, we use the give-up option to classify equilibria.

In A-1-S equilibria either $N_1 = N_2$ (the total number of partitions is even) or $N_1 = N_2 + 1$ (the total number of partitions is odd). To see this, note that by part (ii) of Lemma 2, we only need to rule out the case that $N_2 = N_1 + 1$. Since two agents' messages have alternating rankings and agent 1's lowest message has the lowest ranking, if $N_2 = N_1 + 1$ then the two highest messages of agent 2 must have the highest rankings, which means that they are outcome equivalent. Correspondingly, in A-2-S equilibria either $N_1 = N_2$, or $N_2 = N_1 + 1$. To summarize, there are four types of asymmetric equilibria: A-1-S equilibria with even (total) number of partitions, A-1-S equilibria with odd number of partitions, A-2-S equilibria with even number of partitions, and A-2-S equilibria with odd number of partitions.

In this subsection we only study A-1-S equilibria with even number of partitions in detail, and other types of equilibria will be studied in detail later. Consider an A-1-S equilibrium with $N_1 = N_2 = N$. Suppose agent 1's realized state is $a_{1,n}$, $1 \leq n < N$. Agent 1 should be indifferent between sending message m_1^n and message m_1^{n+1}. The indifference condition is written as:

$$a_{1,n} + b_1 = \frac{1}{2}(a_{2,n} + a_{2,n+1}). \tag{6}$$

For agent 1, whether sending m_1^n or m_1^{n+1} matters (which project will be implemented) only when agent 2's message is m_2^n. In that case, sending message m_1^{n+1} leads to project 1 being implemented and agent 1's payoff is $a_{1,n} + b_1$, and sending message m_1^n leads to project 2 being implemented and agent 1's payoff is $(a_{2,n} + a_{2,n+1})/2$. Similarly, the indifference condition that characterizes $a_{2,n}$, $1 \le n < N$, can be written as:

$$a_{2,n} + b_2 = \frac{1}{2}(a_{1,n} + a_{1,n+1}). \tag{7}$$

Applying (6) and (7) recursively, we get

$$\begin{aligned}
(a_{1,n+1} - a_{1,n}) - (a_{1,n} - a_{1,n-1}) &= 4b_1 + 4b_2 \quad \text{for } 1 < n \le N-1, \\
(a_{2,n+1} - a_{2,n}) - (a_{2,n} - a_{2,n-1}) &= 4b_1 + 4b_2 \quad \text{for } 1 \le n < N-1. \\
a_{1,1} &= \frac{1}{2}a_{2,1} - b_1, \\
a_{2,N-1} &= \frac{1}{2}(a_{1,N-1} + 1) - b_2.
\end{aligned} \tag{8}$$

In (8), the last two difference equations apply to the first and last interior partition points respectively, while the first two difference equations apply to other interior partition points.[9] Note that for most interior partition points, the difference equations for agent 1 are the same as those for agent 2. This confirms that in equilibrium two agents transmit similar amount of information. Another notable feature is that the incremental step size of each agent, which is $4b_1 + 4b_2$, depends on the biases of both agents. The solution to the above difference equation system is

$$\begin{aligned}
a_{2,1} &= \frac{2 - 4b_2 - 2b_1 - 4(b_1 + b_2)N(N-2)}{2N-1} \\
a_{1,1} &= \frac{1 - 2b_2 - 2Nb_1 - 2(b_1 + b_2)N(N-2)}{2N-1} \\
a_{2,n} &= na_{21} + 2(b_1 + b_2)n(n-1) \quad \text{for } 1 < n \le N-1 \\
a_{1,n} &= \frac{2n-1}{2}a_{21} - b_1 + 2(b_1 + b_2)(n-1)^2 \quad \text{for } 1 < n \le N-1
\end{aligned} \tag{9}$$

Example 1 $b_1 = 0.02$ *and* $b_2 = 0.05$. *The most informative A-1-S equilibrium, which has 6 total partitions (each agent has 3 partitions), is illustrated in the following figure.*

From Example 1, we see that the alternating ranking structure of two agents' messages implies that two agents' partitions have the following alternating feature: for any interior partition points $a_{i,n}$, we either have $a_{i,n} \in (a_{j,n-1}, a_{j,n})$ for all n, or have $a_{i,n} \in (a_{j,n}, a_{j,n+1})$. The following proposition summarizes the results we derived so far.

Proposition 2 *There are four types of asymmetric equilibria. In A-1-S equilibrium with even number of partitions,* $N_1 = N_2$*, and in A-1-S equilibrium with odd number of partitions,* $N_1 = N_2+$

[9] The A-1-S equilibria with $N_1 = N_2 + 1$ is qualitatively similar to those with $N_1 = N_2$. In particular, the difference equation applying to the last interior partition point is different, while all the other difference equations are exactly the same.

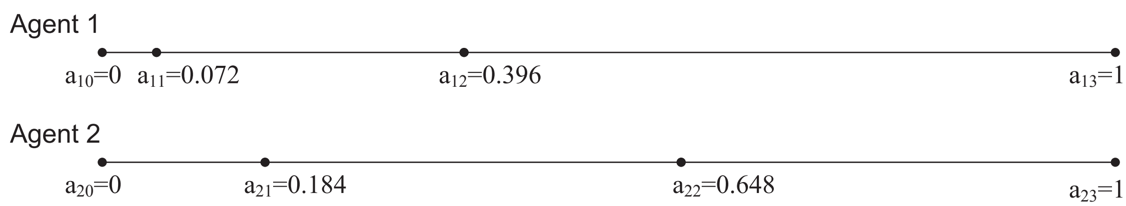

Figure 1: A-1-S odd number partitions

1*; and in both types of equilibria, two agents' partitions have the following alternating feature:* $a_{1,n} \in (a_{2,n-1}, a_{2,n})$ *for all interior* n*, and* $a_{2,n} \in (a_{1,n}, a_{1,n+1})$ *for all interior* n*. In A-2-S equilibrium with even number of partitions,* $N_1 = N_2$*, and in A-2-S equilibrium with odd number of partitions* $N_2 = N_1 + 1$*; and in both types of equilibria, two agents' partitions have the following alternating feature:* $a_{1,n} \in (a_{2,n}, a_{2,n+1})$ *for all interior* n*, and* $a_{2,n} \in (a_{1,n-1}, a_{1,n})$ *for all interior* n*.*

Comparing the difference equations for the interior partition points, one can see that the partition sizes increase a lot faster for asymmetric equilibria with asymmetric agents (the incremental step size is $4b_1 + 4b_2$) than symmetric equilibria with symmetric agents (the incremental step size is $2b$). Thus one would expect that symmetric equilibria with symmetric agents are much more informative. However, it turns out that it is not the case. To illustrate the link between these two types of equilibria, we reformulate asymmetric equilibria by introducing some reducible messages.

4.2 Quasi-symmetric equilibria

We first provide a definition of *quasi-symmetric (pure strategy) equilibria* (QSE).

Definition 3 *QSE are equilibria with the following properties: two agents have the same partitions (hence the same set of messages) and the DM implements one of the projects with probability 1 whenever two agents send the same message.*

Since both agents have the same partitions, we let $N \geq 2$ be the number of partitions and $\{a_n\}$ be the partition points. In the case that both agents send the same message m^n (there is a tie), denote the probability that agent 1 (2)'s project being adopted as λ_n $(1-\lambda_n)$. Since we are talking about pure strategy, λ_n is either 0 or 1.

Lemma 3 *There is no QSE in which* $\lambda_n = \lambda_{n+1} = 0$ *or* $\lambda_n = \lambda_{n+1} = 1$*.*

Proof. Since two agents' situations are symmetric, we only need to rule out the case of $\lambda_n = \lambda_{n+1} = 0$ as an equilibrium. For agent 1 whose type is a_n, he should be indifferent between sending messages m^n and m^{n+1}. Given that $\lambda_n = \lambda_{n+1} = 0$, whether agent 1 sends messages m^n or m^{n+1} matters for the outcome only if agent 2's message is m^n. In this case, if agent 1 sends message

m^{n+1}, then his expected payoff is $a_n + b$. If agent 1 sends message m^n, then his expected payoff is $(a_{n-1} + a_n)/2$, which is strictly less than $a_n + b$. Thus the type a_n of agent 1 cannot be indifferent and it cannot be an equilibrium. ■

Intuitively, if for two adjacent messages the DM's tie-breaking rule always favors the same agent by implementing his project with probability 1, then the marginal type of the disfavored agent cannot be indifferent between sending two messages, since there is no cost of overstating for him. By Lemma 3, in equilibrium for two adjacent messages the principal's tie-breaking rule must favor two agents alternatingly. That is, if $\lambda_n = 0$ then $\lambda_{n+1} = 1$; and if $\lambda_n = 1$ then $\lambda_{n+1} = 0$. For this *alternatingly favored tie-breaking rule*, there are two possibilities: the first one starts with $\lambda_1 = 0$ and the second one starts with $\lambda_1 = 1$. We call the first kind *agent-1-sacrificing (A-1-S) QSE* and the second kind *agent-2-sacrificing (A-2-S) QSE*.

We first investigate A-1-S QSE. More explicitly, for odd $n = 2k+1$, $\lambda_n = 0$, and for even $n = 2k$, $\lambda_n = 1$.[10] Under these tie-breaking rules, note that for any odd (interior) partition points, agent 2's indifference condition is always satisfied, while for any even (interior) partition points agent 1's indifference condition is always satisfied. To see this, pick an odd (interior) partition point, a_n, $n = 2k+1$. Suppose agent 2's type is a_n and consider the scenario that agent 1's message is either m^n or m^{n+1}. Since $\lambda_n = 0$ and $\lambda_{n+1} = 1$, whether project 2 will be implemented does not depend on whether agent 2 sends message m^n or m^{n+1}: if agent 1 sends message m^{n+1} then project 1 is implemented for sure, and if agent 1 sends message m^n then project 2 is implemented for sure. Therefore, agent 2 is indifferent between sending messages m^n and m^{n+1}. Actually, for any type of agent 2 within the interval (a_{n-1}, a_{n+1}), he is indifferent between sending messages m^n and m^{n+1}. Similar logic applies to agent 1 for even (interior) partition points.

Now there are two sets of equilibrium conditions left: agent 1's indifference conditions at odd (interior) partition points, and agent 2's indifference conditions at even (interior) partition points. They can be explicitly written as

$$\begin{aligned} \text{For } n &= 2k+1\text{: } (a_{n+1} - a_{n-1})(a_n + b_1) = (a_{n+1} - a_{n-1})\frac{a_{n+1} + a_{n-1}}{2}, \\ \text{For } n &= 2k\text{: } (a_{n+1} - a_{n-1})(a_n + b_2) = (a_{n+1} - a_{n-1})\frac{a_{n+1} + a_{n-1}}{2}. \end{aligned} \tag{10}$$

The above conditions can be simplified as

$$\begin{aligned} \text{For } n &= 2k+1\text{: } (a_{n+1} - a_n) - (a_n - a_{n-1}) = 2b_1, \\ \text{For } n &= 2k\text{: } (a_{n+1} - a_n) - (a_n - a_{n-1}) = 2b_2. \end{aligned} \tag{11}$$

A-2-S QSE can be characterized in a similar way. In particular, for odd $n = 2k+1$, $\lambda_n = 1$, and for even $n = 2k$, $\lambda_n = 0$. The binding equilibrium conditions are: agent 1's indifference conditions at even (interior) partition points, and agent 2's indifference conditions at odd (interior) partition

[10] Basically, the priority of two projects whenever there is a tie switches alternatingly across messages, starting with project 2 having priority when both agents send the lowest message.

points. More specifically, the difference equations are as follows

$$\begin{aligned}\text{For } n &= 2k\text{: } (a_{n+1}-a_n)-(a_n-a_{n-1})=2b_1, \\ \text{For } n &= 2k+1\text{: } (a_{n+1}-a_n)-(a_n-a_{n-1})=2b_2.\end{aligned} \tag{12}$$

Note that under QSE agents' messages are not irreducible. To see this, consider an A-1-S QSE with 3 partitions (messages) for each agent. For agent 1, the two highest messages are outcome equivalent[11], and for agent 2 the two lowest messages are outcome equivalent. If we combine all outcome equivalent messages for each agent, then a QSE becomes an asymmetric equilibrium. The following lemma shows that there is a one-to-one correspondence between asymmetric equilibria and QSE.

Lemma 4 *(i) For any A-i-S asymmetric equilibrium with partition points* (a_1, a_2)*, combine all interior points of* a_1 *and* a_2 *and the two boundary points of* 0 *and* 1*, and rearrange them into an increasing sequence* $a\prime$*. Then there is an A-i-S QSE with partition points* a'*. (ii) For any A-i-S QSE with partition points* a*, combine all outcome equivalent messages for each agent, and let the resulting partition points be* (a'_1, a'_2)*. Then there is an A-i-S asymmetric equilibrium with partition points* (a'_1, a'_2)*.*

Proof. We only need to show the claims hold for A-1-S equilibria, since the situations for A-2-S equilibria are similar.

Part (i). Consider an A-1-S asymmetric equilibrium with $N_1 = N_2 = N$, and the partition points being (a_1, a_2). For partition points a' constructed from (a_1, a_2) according to the procedure stated in the lemma, a' has $2N-2$ interior partition points. We argue that there is an A-1-S QSE with a' being each agent's partition points. In particular, each agent has $2N-1$ partitions. Specifically, for odd n', interior partition point $a'_{n'} = a_{1,\frac{n'+1}{2}}$; and for even n', interior partition point $a'_{n'} = a_{2,\frac{n'}{2}}$. Observing the indifference conditions, we see that (6) and (7) for the asymmetric equilibrium are exactly the same as (10) for QSE. Therefore, there is an A-1-S QSE with partition points a'.

By similar logic, we can show that for an A-1-S asymmetric equilibrium with $N_1 = N$ and $N_2 = N-1$, there is a corresponding QSE with $2N-2$ partitions for each agent.

Part (ii). The opposite direction can be proved in a similar way, since the indifference conditions for two kinds of equilibria are essentially the same. ■

By Lemma 4, the set of asymmetric equilibria and the set of QSE are essentially the same. Although our focus is on asymmetric equilibria, we will solve for QSE since they are easier to characterize.

Example 2 *The following figure illustrates the QSE corresponding to the A-1-S asymmetric equilibrium in Example 1. In the QSE both agents have 5 partitions, and the brackets indicate how to group these partitions into the partitions of asymmetric equilibrium.*

[11] As long as agent 1 sends one of his two highest messages, if agent 2 sends his highest message then project 2 is always implemented, and otherwise project 1 is always implemented.

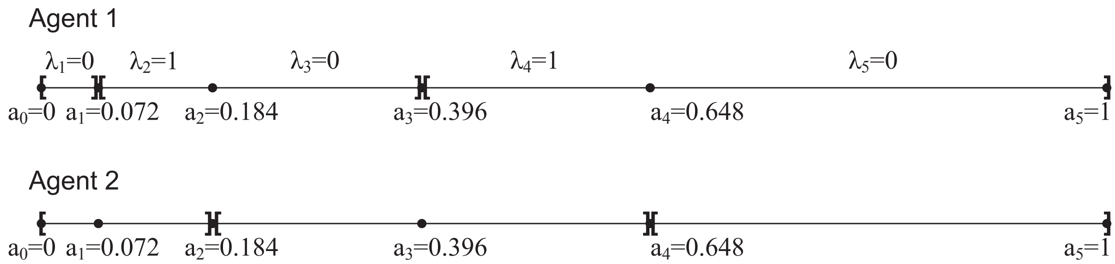

Figure 2: The Equivalence between QSE and Asymmetric Equilibrium

Based on the proof of Lemma 4, we summarize the correspondence between asymmetric equilibria and QSE in the following table.

Table 1: The Correspondence between Asymmetric Equilibria and QSE

Asymmetric equilibria	Corresponding QSE
A-1-S with $2N$ partitions	A-1-S with $2N-1$ partitions
A-1-S with $2N-1$ partitions	A-1-S with $2N-2$ partitions
A-2-S with $2N$ partitions	A-2-S with $2N-1$ partitions
A-2-S with $2N-1$ partitions	A-2-S with $2N-2$ partitions

Now we characterize QSE. For A-1-S QSE, solving the difference equations of (11), for even N we have

$$a_1 = \frac{1}{N} - \frac{N}{2}b_1 - (\frac{N}{2}-1)b_2.$$

The upper bound of N, denoted as $\overline{N}_E^{A1S}$, is the largest even integer N that satisfies the following inequality:

$$\frac{N^2}{2}b_1 + N(\frac{N}{2}-1)b_2 < 1. \tag{13}$$

Similarly, for odd N the difference equations of (11) yield

$$a_1 = \frac{2-(N-1)(N+1)b_1-(N-1)^2b_2}{2N}. \tag{14}$$

The upper bound of N, denoted as $\overline{N}_O^{A1S}$, is the largest odd integer N that satisfies the following inequality

$$\frac{(N-1)(N+1)}{2}b_1 + \frac{(N-1)^2}{2}b_2 < 1. \tag{15}$$

The largest number of equilibrium partitions, denoted as $\overline{N}^{A1S}$, can be simply found as $\overline{N}^{A1S} = \max\{\overline{N}_E^{A1S}, \overline{N}_O^{A1S}\}$.

Just like other cheap talk models, there are multiple equilibria even for A-1-S equilibria. Specifically, given $\overline{N}^{A1S}$, for any integer N such that $1 \leq N < \overline{N}^{A1S}$, there is an A-1-S equilibrium with N partitions. This is because, for such an N, (13) is satisfied if N is even and (15) is satisfied if N is odd. Following the cheap talk literature, our main focus will be on the most informative equilibrium.

The equilibrium expected payoff of the DM, $E(U_p(N))$, can be written as:

$$E(U_p^{A1S}(N)) = \frac{1}{2}\sum_{n=1}^{N}[(a_n - a_{n-1})a_n(a_n + a_{n-1}) + (a_n - a_{n-1})(1 - a_n^2)],$$

which can be explicitly calculated as

$$E(U_p^{A1S}(N)) = \begin{cases} \frac{2}{3} - \frac{1}{6N^2} - \frac{\frac{(b_1+b_2)^2}{4}N^2 - (b_1+b_2)^2 + 3b_1^2}{6} & N \text{ even} \\ \frac{2}{3} + \frac{(b_2-b_1-1)b_1^2 + (4-b_2)b_1b_2 + (b_2-1)b_2^2}{12} + \frac{b_1-b_2-1}{24}[\frac{(b_1-b_2+2)^2}{N^2} + (b_1+b_2)^2N^2] & N \text{ odd} \end{cases}. \tag{16}$$

From the expression of (16), it is easy to verify that the equilibrium expected payoff of the DM is increasing in N (for $N < \overline{N}^{A1S}$), and decreasing in both b_1 and b_2. In QSE, the loss of the DM's expected payoff is again due to the possibility of tie: when two agents send the same message, one project is chosen while the other one could have a higher return. Note that when $b_1 = b_2$, the partitions in QSE become the same as those in the symmetric equilibrium with symmetric agents, though the tie-breaking rules in two equilibria are still different. Also, when $b_1 = b_2$, the expected payoff of the DM under QSE, $E(U_p^{A1S}(N))$, and that under the symmetric equilibrium, $E(U_p(N))$, are the same.[12]

The characterization of A-2-S QSE is very similar to that of A-1-S QSE, with the role of two agents (or b_1 and b_2) being switched. Specifically, let $\overline{N}_E^{A2S}$ be the largest even integer N that satisfies the following inequality:

$$\frac{N^2}{2}b_2 + N(\frac{N}{2} - 1)b_1 < 1. \tag{17}$$

And let $\overline{N}_O^{A2S}$ be the largest odd integer N that satisfies the following inequality

$$\frac{(N-1)(N+1)}{2}b_2 + \frac{(N-1)^2}{2}b_1 < 1. \tag{18}$$

Moreover, the largest number of equilibrium partitions, denoted as $\overline{N}^{A2S}$, can be simply found as $\overline{N}^{A2S} = \max\{\overline{N}_E^{A2S}, \overline{N}_O^{A2S}\}$. Finally, the DM's expected payoff is

[12] As long as the partitions are the same, the tie-breaking rules do not affect the DM's expected payoff since two projects have the same expected return conditional on the same message sent.

$$E(U_p^{A2S}(N)) = \begin{cases} \frac{2}{3} - \frac{1}{6N^2} - \frac{\frac{(b_1+b_2)^2}{4}N^2-(b_1+b_2)^2+3b_2^2}{6} & N \text{ even} \\ \frac{2}{3} + \frac{(b_1-b_2-1)b_2^2+(4-b_1)b_1b_2+(b_1-1)b_1^2}{12} + \frac{b_2-b_1-1}{24}[\frac{(b_2-b_1+2)^2}{N^2} + (b_1+b_2)^2N^2] & N \text{ odd} \end{cases}. \tag{19}$$

Since our focus is the most informative equilibrium, now we study the following question: under what circumstances the most informative QSE is an A-1-S QSE? In other words, in the most informative equilibrium should the agent of a smaller bias always have the give-up option? To proceed, the next lemma compares A-1-S QSE and A-2-S QSE.

Lemma 5 *(i) The number of partitions in the most informative A-1-S QSE is weakly larger than that of the most informative A-2-S QSE:* $\overline{N}^{A2S} \leq \overline{N}^{A1S} \leq \overline{N}^{A2S} + 1$. *(ii) For equilibria with the same number of partitions N, with N being even, an A-1-S QSE is more informative (leads to a higher $E(U_p(N))$) than an A-2-S QSE. (iii) For equilibria with the same number of partitions $N > 1$, with N being odd, an A-2-S QSE is more informative than an A-1-S QSE.*

Proof. Part (i). Inspecting (13) and (17), for the same even N we can see that the LHS of the inequality is larger under an A-2-S QSE than under an A-1-S QSE, since $b_1 < b_2$. By (15) and (18), the same pattern holds for odd N. Therefore, we must have $\overline{N}^{A2S} \leq \overline{N}^{A1S}$. To show that $\overline{N}^{A1S} \leq \overline{N}^{A2S} + 1$, first consider the case that N is even. Note that the LHS of (13) with N is larger than the LHS of (18) with $N-1$. Thus, if $\overline{N}^{A1S}$ is even, then $\overline{N}^{A1S} \leq \overline{N}^{A2S} + 1$. When N is odd, it can be verified that the LHS of (15) with N is larger than the LHS of (17) with $N-1$. Thus, if $\overline{N}^{A1S}$ is odd, then $\overline{N}^{A1S} \leq \overline{N}^{A2S} + 1$.

Part (ii). Consider an A-1-S QSE and an A-2-S QSE with the same even N. By (16) and (19), for even N, $E(U_p^{A1S}(N)) - E(U_p^{A2S}(N)) = (b_2^2 - b_1^2) > 0$. This implies that the A-1-S QSE is more informative than the A-2-S QSE.

Part (iii). Consider an A-1-S QSE and an A-2-S QSE with the same odd N. By (16) and (19), for odd N, we have

$$\begin{aligned} E(U_p^{A1S}(N)) - E(U_p^{A2S}(N)) &\propto 2[(b_2^3 - b_1^3) + b_1b_2(b_1 - b_2)] + \frac{(b_1 - b_2)^3}{N^2} + (b_1 - b_2)(b_1 + b_2)^2N^2 \\ &< 2(b_2^3 - b_1^3) + (b_1 - b_2)(b_1 + b_2)^2N^2 < 0, \end{aligned}$$

where the last inequality uses the fact that $N \geq 3$ (informative equilibrium). Therefore, the A-2-S QSE is more informative than the A-1-S QSE. ■

To understand the intuition behind Lemma 5, we compare the patterns of partitions between two types of equilibria. Specifically, let $\{a_n\}$ and $\{a'_n\}$ be the sequence of partition points, and let the size of n^{th} partition be $a_1 + \Delta_n$ and $a'_1 + \Delta'_n$ ($\Delta_1 = \Delta'_1 = 0$), for A-1-S QSE and A-2-S QSE, respectively. The term Δ_n can be interpreted as the incremental partition size of the nth partition relative to the size of the first partition. By the difference equations (11), for A-1-S QSE Δ_n follows the following pattern: 0, $2b_1$, $2b_1 + 2b_2$, $4b_1 + 2b_2$, $4b_1 + 4b_2$, ... By the difference equations (11), for A-2-S QSE Δ'_n follows the following pattern: 0, $2b_2$, $2b_1 + 2b_2$, $2b_1 + 4b_2$, $4b_1 + 4b_2$, ... From these patterns we can see that, in A-1-S QSE b_1 enters into the incremental step size more often than b_2 does, while in A-2-S QSE it is the opposite. This implies that, compared to A-2-S QSE, in A-1-S QSE the partition sizes increase more slowly, which potentially allows more partitions.

The partition patterns imply that, for odd n we have $\Delta_n = \Delta'_n$, and for even n we have $\Delta'_n - \Delta_n = 2(b_2 - b_1) > 0$. We first consider the case that both A-1-S QSE and A-2-S QSE have the same even number, N, of partitions. By the fact that the total length of all partitions must be 1, we have

$$N(a_1 - a'_1) + \sum_{n=1}^{N} (\Delta_n - \Delta'_n) = 0. \tag{20}$$

Since N is even, (20) implies that $a_1 - a'_1 = b_2 - b_1 > 0$. For $1 < n < N$, we have

$$a_n - a'_n = n(a_1 - a'_1) + \sum_{j=1}^{n} (\Delta_j - \Delta'_j). \tag{21}$$

Using the fact that $a_1 - a'_1 = b_2 - b_1$ and the cyclical pattern of $\Delta_j - \Delta'_j$, we conclude that, for n odd $a_n > a'_n$, and for n even $a_n = a'_n$. Given this pattern, on average A-1-S QSE leads to relatively more even partitions. Recall that more even partitions reduce the ex ante probability of tie (two agents send the same message), which is the soruce of inefficiency. Therefore, A-1-S QSE results in a higher expected payoff for the DM.

Example 3 $b_1 = 0.06$, $b_2 = 0.08$. *The most informative A-1-S QSE and A-2-S QSE are illustrated in the following figure. Both equilibria have 4 partitions. The partition points a_2 are the same under two equilibria, but a_1 and a_3 are bigger under A-1-S QSE than those under A-2-S QSE. Therefore, overall the partition under A-1-S QSE is more even.*

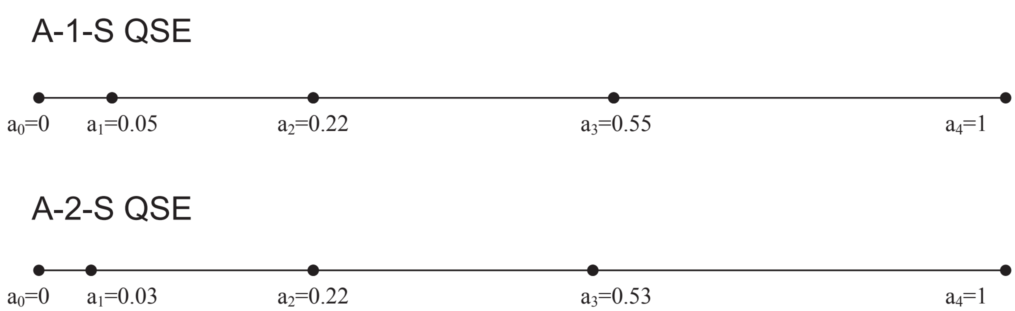

Figure 3: A-1-S QSE Has More Even Partitions

Now consider the case that both A-1-S QSE and A-2-S QSE have the same odd number, N, of partitions. Since N is odd, (20) implies that $a_1 - a'_1 = (b_2 - b_1)(N-1)/N$. Now by (21), for odd n we have:

$$a_n - a'_n = n\frac{N-1}{N}(b_2 - b_1) - (n-1)(b_2 - b_1) > 0.$$

And for even n we have:

$$a_n - a'_n = n\frac{N-1}{N}(b_2 - b_1) - n(b_2 - b_1) < 0.$$

The above inequalities indicate the following pattern. For two adjacent partitions starting with an odd partition, the partitions under A-1-S QSE are more even. However, for two adjacent partitions starting with an even partition, the partitions under A-2-S QSE are more even. Since the total number of partitions is odd, for the last two partitions the partitions under A-2-S QSE are more even. Because the partitions are increasing in size, making larger partitions more even are more important.[13] Therefore, A-2-S QSE leads to more even partitions and is more informative overall.

Example 4 $b_1 = 0.1$, $b_2 = 0.16$. *The most informative A-1-S QSE and A-2-S QSE are illustrated in the following figure. Both equilibria have 3 partitions. Compared to the A-1-S QSE, for the A-2-S QSE, though the first partition size is smaller, the sizes of the second and third partitions are closer, which leads to more even partitions overall. In particular,* $E(U_p^{A2S}) = 0.626559$, *which is greater than* $E(U_p^{A1S}) = 0.623871$.

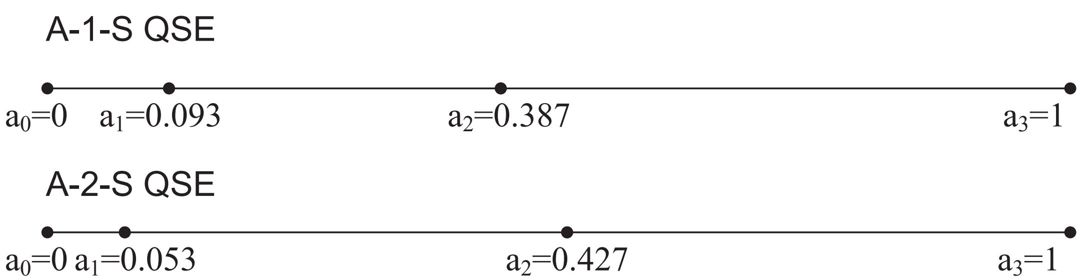

Figure 4: A-1-S QSE and A-2-S QSE

The following lemma identifies conditions under which the most informative equilibrium is A-1-S QSE or A-2-S QSE.

Lemma 6 *(i) If the most informative A-1-S QSE has an even number of partitions, then it must be the most informative QSE. (ii) Suppose the most informative A-1-S QSE has an odd number of partitions, and the most informative A-2-S QSE has a smaller number of partitions. Then the most informative QSE must be the most informative A-1-S QSE. (iii) Suppose the most informative A-1-S QSE and the most informative A-2-S QSE have the same odd number of partitions, then the most informative QSE must be the most informative A-2-S QSE.*

[13]The ex ante probability of tie depends more heavily on the size of the largest partition. That is why making the larger partitions more even, which reduces the size of the largest partition, is more important.

Proof. Part (i). Suppose the most informative A-1-S QSE has an even number of partitions, say N. By Part (i) of Lemma 5, the number of partitions in the most informative A-2-S QSE can be either N or $N-1$. In the first case, the most informative A-1-S QSE is the most informative QSE by part (ii) of Lemma 5. In the latter case, the most informative A-1-S QSE is obviously more informative than the most informative A-2-S QSE.

Part (ii). Let N, which is odd, be the number of partitions in the most informative A-1-S QSE. Consider the case that the most informative A-2-S QSE has $N-1$ partitions. By previous results, we have $E(U_p^{A1S}(N)) - E(U_p^{A1S}(N-1)) > 0$. Since $N-1$ is even, by part (ii) of Lemma 5, we have $E(U_p^{A1S}(N-1)) - E(U_p^{A2S}(N-1)) > 0$. Combining the above two inequalities, we conclude that $E(U_p^{A1S}(N)) - E(U_p^{A2S}(N-1)) > 0$. Therefore, the most informative A-1-S QSE is the most informative QSE.

Part (iii). It directly follows part (iii) of Lemma 5. ■

4.3 Properties of asymmetric equilibria

Now we go back to examine the properties of asymmetric equilibria. One property worth emphasizing is that for the agent who has the highest overall message, his highest partition might be smaller than his second highest partition. Recall example 1, in which agent 2 has the highest overall message. The size of agent 2's highest partition (the 3rd partition) is 0.352, which is smaller than the size of the second highest partition (2nd partition), 0.464.[14] This is very different from standard cheap talk models, in which the sizes of partitions are always increasing in the direction of agents' biases. This property is due to the competitive nature of cheap talk. For the highest marginal type of the agent who has the highest overall message, to make that type indifferent between sending two adjacent messages, only the size of the highest partition of the other agent matters and the size of the highest partition of himself does not matter.

In asymmetric equilibria, since there is no possibility of tie of messages, the source of inefficiency is that two agents' adjacent partitions overlap. This means that conditional on two agents send two adjacent messages, while one agent's message have a higher posterior (ranking) and thus this agent's project getting implemented, the other agent's realized return might be higher. Translating asymmetric equilibria to corresponding QSE, the inefficiency due to overlapping partitions is equivalent to the inefficiency due to ties when additional messages are introduced.

Corollary 1 *Fixing b_i, the DM's expected payoff in the most informative equilibrium is decreasing in b_j.*

Proof. Given the symmetry of two agents' situations, we only need to show that the claim holds for agent 2. Fix b_1, and suppose b_2 decreases to $b_2' < b_2$. It is enough to show that the DM's payoff in the most informative A-1-S equilibrium and that in the most informative A-2-S equilibrium both increase. Consider A-1-S equilibria first. Since $b_2' < b_2$, by previous results $\overline{N}^{A1S} \leq \overline{N}'^{A1S}$. If $\overline{N}^{A1S} < \overline{N}'^{A1S}$, then in the most informative equilibrium the DM's payoff must be higher under b_2'. If $\overline{N}^{A1S} = \overline{N}'^{A1S}$, by (16), again in the most informative equilibrium the DM's payoff is higher

[14] It can be verified that, for the agent who does not have the highest overall message, the partition sizes are monotonically increasing. And for the agent who has the highest overall message, the partition sizes are always monotonically increasing up to the second highest message.

under b_2'. Similarly, one can show that the DM's payoff in the most informative A-2-S equilibrium is higher under b_2'. ■

A result stronger than Corollary 1 holds: both agents will transmit more information in the most informative equilibrium if one agent's bias decreases. Thus in some sense two agents' information transmissions are strategic complements.[15] This feature is also present in the two-sender cheap talk model of McGee and Yang (2013), but for a different reason. The reason for this property to arise in the current model is again due to the competitive nature of cheap talk. Intuitively, one agent will exaggerate less and transmit more information if he has a smaller bias. As mentioned earlier, since only the comparison between two agents' projects matters, the finer information transmitted by one agent allows the other agent to be able to transmit more meaningful information.

Now we study under what conditions the most informative asymmetric equilibrium is an A-1-S equilibrium, or agent 1 has the give-up option. Given the correspondence between asymmetric equilibria and QSE, the following proposition directly follows Lemma 5 and Lemma 6.

Proposition 3 *(i) Suppose the total number of partitions in the most informative A-2-S asymmetric equilibrium is N, then the total number of partitions in the most informative A-1-S asymmetric equilibrium is either N or $N+1$. (ii) If in the most informative A-1-S asymmetric equilibrium the total number of partitions is odd, then the most informative A-1-S equilibrium is the most informative equilibrium. (iii) If in the most informative A-1-S asymmetric equilibrium the total number of partitions is even, say $2N$, and in the most informative A-2-S asymmetric equilibrium the total number of partitions is $2N-1$, then the most informative A-1-S equilibrium is the most informative equilibrium. (iv) If the most informative A-1-S asymmetric equilibrium and the most informative A-2-S asymmetric equilibrium have the same total even number of partitions, then the most informative A-2-S equilibrium is the most informative equilibrium.*

Proposition 3 directly implies the following corollary.

Corollary 2 *In the most informative equilibrium: (i) it is not always the case, but it is more likely, that the agent with a smaller bias has the give-up option; (ii) it is not always the case, but it is more likely, that the agent with a smaller bias has the sure option; (iii) relative to the agent who has a bigger bias, the agent with a smaller bias either has the same number of messages or has one more message; (iv) while it is possible for the agent with a smaller bias to have the give-up option and sure option at the same time, it is impossible for the agent with a bigger bias to have both options at the same time.*

Proof. By Proposition 3, the most informative equilibrium could be one of the following three equilibria: A-1-S equilibrium with odd number of partitions, A-1-S equilibrium with even number of partitions, A-2-S equilibrium with even number of partitions. Agent 1 has the give-up option in two out of three scenarios, thus he is more likely to have the give-up option. In terms of the sure option, agent 1 has the sure option in the first type and third type of equilibria, while agent 2 has the sure option in the second type of equilibria. Therefore, it is more likely for agent 1 to have the

[15] The technical reason is that, as mentioned earlier (euqations (9)), the incremental step size of the interior partitions for each agent is $4b_1+4b_2$. This implies that, when one agent's bias decreases, then in the most informative equilibrium the other agent's number of partitions will weakly increase and his partitions will become more even.

sure option. This proves parts (i) and (ii). As to part (iii), note that in the first type of equilibria agent 1 has one more message than agent 2 does, while in other two types of equilibria both agents have the same number of messages. Regarding part (iv), in the first type of equilibria agent 1 has both the give-up option and the sure option, while in the second and third types of equilibria two agents split the give-up option and the sure option. ■

The agent having the give-up option can be interpreted as being trustworthy for (ruling out) low return projects, and the agent having the sure option being trustworthy for high return projects. Note that being trustworthy is endogenous. With these interpretations, Corollary 2 implies that in the most informative equilibrium the agent with a smaller bias is chosen to be trustworthy more often not only for low return projects but also for high return projects. Actually, the agent with a smaller bias must be trustworthy for at least one end (either the low end, or the high end, or both). Moreover, while the agent with a smaller bias could be trustworthy for both low return projects and high return projects at the same time, it is never the case for the agent with a bigger bias. Part (iii) of Corollary 2 indicates that in the most informative equilibrium the agent with a smaller bias has weakly more messages or partitions.[16] These predictions are potentially testable.

In the following figure, quantitatively we illustrate the frequency of each type of equilibrium being the most informative equilibrium. Specifically, the blue (yellow, red) areas are the combinations of the biases such that an A-1-S equilibrium with even number of partitions (A-1-S equilibrium with odd number of partitions, A-2-S equilibrium with even number of partitions) is the most informative equilibrium. The figure shows that overall the most informative equilibrium is more likely to be an A-1-S equilibrium, as the blue and yellow areas are significantly bigger than the red areas.

The most informative equilibrium might not be Pareto dominant: while it is clear that the DM always prefers the most informative equilibrium, the two agents might prefer different equilibria as they also take into account the probabilities that their own projects will be implemented. The ex ante probabilities that each project will be implemented in different equilibria are characterized in the following Proposition.

Proposition 4 *(i) In A-i-S asymmetric equilibrium with the total number of partitions being odd, the ex ante probability that project i (j) is implemented is strictly greater (less) than* $1/2$*. (ii) In A-i-S asymmetric equilibrium with the total number of partitions being even, the ex ante probability that project i (j) is implemented is strictly less (greater) than* $1/2$*.*

Proof. Since the situations of A-1-S and A-2-S equilibria are similar, we only prove the claims for A-1-S equilibria.

Part (i). Consider a corresponding A-1-S QSE with an even number (say $2N$) of partitions. Since the returns of the two projects have the same distribution, the probability that θ_1 lies in a higher partition than θ_2 does is the same as the probability that θ_2 lies in a higher partition than θ_1 does. Therefore, we only need to consider the situations that both θ_1 and θ_2 lie in the same partition (or ties). Recall that the alternating tie-breaking rule favors agent 2 for $(2n-1)th$ partition, and favors agent 1 for $(2n)th$ partition. Given that in total there are $2N$ partitions,

[16] However, in the most informative equilibrium the agent with a smaller bias could transmit less amount of information than the other agent does. When the most informative equilibrium is an A-1-S equilibrium with even number of partitions, agent 2's partitions are more even and hence he transmits more information than agent 1 does.

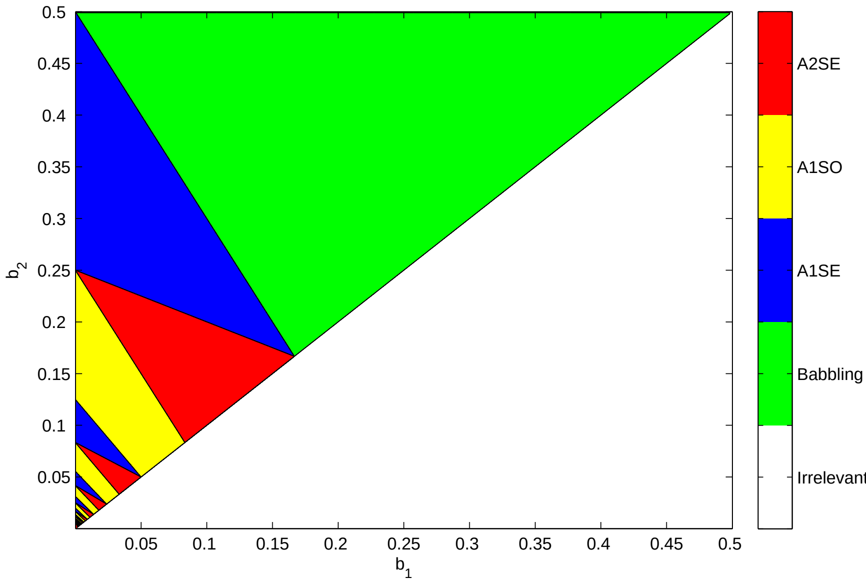

Figure 5: The Most Informative Equilibrium and the Biases

we can group all $2N$ partitions into N pairs, with each pair containing two adjacent partitions: $(2n-1)th$ partition and $(2n)th$ partition. Since the partition sizes are increasing, ties for higher partitions are more likely. This implies that for each pair of partitions, project 1 is more likely to be implemented than project 2 is. Therefore, overall project 1 (2) will be implemented with a probability strictly greater (less) than 1/2.

Part (ii). Consider a corresponding A-1-S QSE with an odd number (say $2N+1$) of partitions. The proof is similar to that of part (i). The only difference is that we need to use different grouping. Given that in total there are $2N+1$ partitions, we can group the $2N$ highest partitions into N pairs, with each pair containing two adjacent partitions: $(2n)th$ partition and $(2n+1)th$ partition. Since the partition sizes are increasing, ties for higher partitions are more likely. This implies that for each pair of partitions, project 2 is more likely to be implemented than project 1 is. Moreover, in the 1st partition project 2 is favored. Therefore, overall project 2 (1) will be implemented with a probability strictly greater (less) than 1/2. ∎

The results of Proposition 4 can be restated in a more compact way. In any equilibrium, the project of the agent who has the sure option (the highest overall message) always has an ex ante probability bigger than 1/2 of being implemented. Therefore, all other things being equal, each agent prefers to have the sure option. The intuition for this result is that in QSE the partition sizes are monotonically increasing. Given that the agent who has the sure option is favored in the highest partition, and the tie-breaking rule is alternatingly favored, the project of the agent who has the sure option is favored in larger combined intervals.

Since the most informative equilibrium might not be Pareto dominant, we cannot invoke Pareto dominance to select the most informative equilibrium. However, we argue that more informative equilibria are the more reasonable ones, based on equilibrium refinement by introducing out of equilibrium messages. The details of the equilibrium refinement can be found in the Appendix.

Is the DM able to select A-1-S equilibria or A-2-S equilibria? The answer is yes if the DM is able to commit. For example, suppose the DM wants to select A-1-S equilibria. To achieve that, the DM can commit to the following: if both agents send the lowest messages, then project 2 will be implemented. Given this commitment, A-2-S equilibria will no longer be equilibrium as the indifference conditions are messed up, but A-1-S equilibria are not affected and remain as equilibrium.

4.4 Comparative Statics

In this subsection we study the following question: fixing the total bias of two agents ($b_1 + b_2$), does the DM prefer two agents having relatively equal biases or relatively unequal biases? For that purpose, we fix $b_1 + b_2 = 2b$, and let $b_2 - b_1 = 2d$ be the difference of the biases, $0 \leq d \leq b$. Note that $b_2 = b + d$ and $b_1 = b - d$. As d increases, two agents' biases become further apart. We are interested in how the DM's expected payoff in the most informative equilibrium will change as d changes. Specifically, consider $d' > d$. And we use superscript $'$ to denote the endogenous variables under d'.

Lemma 7 *(i) For A-1-S asymmetric equilibrium, either* $\overline{N}^{A1S'} = \overline{N}^{A1S}$ *or* $\overline{N}^{A1S'} = \overline{N}^{A1S} + 1$. *For A-2-S equilibrium, either* $\overline{N}^{A2S'} = \overline{N}^{A2S}$ *or* $\overline{N}^{A2S'} = \overline{N}^{A2S} - 1$. *(ii) For A-1-S asymmetric equilibrium, if* $\overline{N}^{A1S}$ *(under the initial d) is odd, then* $E(U_p^{A1S'}) > E(U_p^{A1S})$. *If* $\overline{N}^{A1S}$ *is even, then* $E(U_p^{A1S'}) > E(U_p^{A1S})$ *if* $\overline{N}^{A1S'} = \overline{N}^{A1S} + 1$, *and* $E(U_p^{A1S'}) < E(U_p^{A1S})$ *if* $\overline{N}^{A1S'} = \overline{N}^{A1S}$. *(iii) For A-2-S asymmetric equilibrium, if* $\overline{N}^{A2S'} = \overline{N}^{A2S} - 1$, *then* $E(U_p^{A2S'}) < E(U_p^{A2S})$. *If* $\overline{N}^{A2S'} = \overline{N}^{A2S}$ *and* $\overline{N}^{A2S}$ *is odd, then* $E(U_p^{A2S'}) < E(U_p^{A2S})$. *If* $\overline{N}^{A2S'} = \overline{N}^{A2S}$ *and* $\overline{N}^{A2S}$ *is even,* $E(U_p^{A2S'})$ *could either be smaller or bigger than* $E(U_p^{A2S})$.

Proof. For convenience, we translate asymmetric equilibria into the corresponding QSE.

Part (i). Rearrange the inequalities regarding the number of partitions of A-1-S QSE, (13) and (15), we get

$$\begin{aligned} (N^2 - N)b - Nd &< 1 \text{ for even } N, \\ [(N-1)^2 + (N-1)]b - (N-1)d &< 1 \text{ for odd } N. \end{aligned}$$

Since the LHS of the above inequalities is decreasing in d, it follows that $\overline{N}^{A1S'} \geq \overline{N}^{A1S}$. Since $d \leq b$, $\overline{N}^{A1S'} \leq \overline{N}^{A1S} + 1$. Therefore, either $\overline{N}^{A1S'} = \overline{N}^{A1S}$, or $\overline{N}^{A1S'} = \overline{N}^{A1S} + 1$. In similar fashion, we can show that, for A-2-S QSE, either $\overline{N}^{A2S'} = \overline{N}^{A2S}$ or $\overline{N}^{A2S'} = \overline{N}^{A2S} - 1$.

Part (ii). For A-1-S QSE, suppose the initial $\overline{N}^{A1S}$ is even. By part (i) there are two cases to consider. In the first case that $\overline{N}^{A1S'} = \overline{N}^{A1S} + 1$, it is obvious that $E(U_p^{A1S'}) > E(U_p^{A1S})$. In the second case that $\overline{N}^{A1S'} = \overline{N}^{A1S}$, by (16) the only term in $E(U_p^{A1S})$ (for $\overline{N}^{A1S}$ even) that depends on d is $-(b-d)^2/2$, which is increasing in d. Therefore, $E(U_p^{A1S'}) > E(U_p^{A1S})$.

Now suppose the initial $\overline{N}^{A1S}$ is odd. If $\overline{N}^{A1S'} = \overline{N}^{A1S} + 1$, then it is obvious that $E(U_p^{A1S'}) > E(U_p^{A1S})$. Now suppose $\overline{N}^{A1S'} = \overline{N}^{A1S}$. By (16) the only term in $E(U_p^{A1S})$ (for $\overline{N}^{A1S}$ odd) that depends on d is as follows:

$$E(U_O^{A1S}(N)) \propto -(2b^2dN^2 + 3d^2 - 2d^3),$$

which is decreasing in d since $d < 1$. Therefore, $E(U_p^{A1S'}) < E(U_p^{A1S})$.

Part (iii). For A-2-S QSE, by part (i) there are two cases to consider. In the first case that $\overline{N}^{A2S'} = \overline{N}^{A2S} - 1$, it is obvious that $E(U_p^{A2S'}) > E(U_p^{A2S})$. Now consider the second case in which $\overline{N}^{A2S'} = \overline{N}^{A2S}$. Suppose $\overline{N}^{A2S}$ is even. By (19) the only term in $E(U_p^{A2S})$ that depends on d is $-(b+d)^2/2$, which is decreasing in d. Therefore, $E(U_p^{A2S'}) < E(U_p^{A2S})$. Now suppose $\overline{N}^{A2S}$ is odd. By (19) the only term in $E(U_p^{A2S})$ that depends on d is as follows:

$$E(U_O^{A2S}(N)) \propto d(2b^2N^2 - 3d - 2d^2).$$

By this equation, $E(U_p^{A2S})$ increases in d if and only $2b^2(\overline{N}^{A2S})^2 - 6d - 6d^2 > 0$. But the sign of this inequality cannot be determined. ■

To understand the intuition of Lemma 7, first consider A-1-S QSE. Recall that the incremental partition size Δ_n follows the following pattern: 0, $2b_1$, $2b_1 + 2b_2$, $4b_1 + 2b_2$, $4b_1 + 4b_2$, ... We can see that, as two agents' biases become further apart (d increases), while the incremental partition sizes of odd number of partitions do not change, those of even number of partitions decreases since b_1 decreases. Therefore, the maximum number of partitions will either stay the same or increase by 1. If the maximum number of partitions under the initial d is even and it stays the same as d increases, the reason that the DM's expected payoff in the most informative equilibrium increases is as follows. Recall that the DM's expected payoff is increasing if the two largest partitions become more even. When the total number of partitions is even, the difference between the sizes of the two largest partitions is $2b_1$. This means that an increase in d leads to overall more even partitions. For the same reason, when the maximum number of partitions under the initial d is odd and it stays the same as d increases, an increase in d makes the two largest partitions more uneven, leading to a lower expected payoff to the DM.

Now consider A-2-S QSE. Recall that the incremental partition size Δ_n follows the following pattern: 0, $2b_2$, $2b_1 + 2b_2$, $2b_1 + 4b_2$, $4b_1 + 4b_2$, ... We can see that, as d increases, while the incremental partition sizes of even number of partitions do not change, those of odd number of partitions increase since b_2 increases. Therefore, the maximum number of partitions will either stay the same or decrease by 1.

Proposition 5 *(i) If initially the most informative asymmetric equilibrium is A-1-S and* $\overline{N}^{A1S}$ *is odd, then an increase in* d *makes the DM better off in the most informative equilibrium. (ii) If initially the most informative asymmetric equilibrium is A-1-S and* $\overline{N}^{A1S}$ *is even, then an increase in* d *could makes the DM either better off or worse off in the most informative equilibrium. (iii) If initially the most informative asymmetric equilibrium is A-2-S, then an increase in* d *could makes the DM either better off or worse off in the most informative equilibrium.*

Proof. For convenience, we again translate asymmetric equilibria into the corresponding QSE.

Part (i). Let $d' > d$. By part (i) of Lemma 7, either $\overline{N}^{A1S'} = \overline{N}^{A1S} + 1$ or $\overline{N}^{A1S'} = \overline{N}^{A1S}$. In the first case, $\overline{N}^{A1S'} > \overline{N}^{A2S'}$ since $\overline{N}^{A2S'} \leq \overline{N}^{A2S} \leq \overline{N}^{A1S}$. Therefore, the most informative equilibrium is still the most informative A-1-S equilibrium. By part (ii) of Lemma 7, $E(U_p^{A1S'}) > E(U_p^{A1S})$. It follows that under d' the DM is better off in the most informative equilibrium.

Part (ii). Let $d' > d$. If $\overline{N}^{A1S'} = \overline{N}^{A1S} + 1$, since $\overline{N}^{A1S'}$ is even, under d' the most informative equilibrium must be A-1-S. In this case, an increase in d makes the DM better off. If $\overline{N}^{A1S'} = \overline{N}^{A1S}$, we first argue that the most informative equilibrium under d' is still A-1-S. By the fact that under d the most informative equilibrium is A-1-S and $\overline{N}^{A1S}$ is odd, we must have $\overline{N}^{A2S} = \overline{N}^{A1S} - 1$, since otherwise the most informative equilibrium would have been A-2-S. Now by part (i) of Lemma 7, $\overline{N}^{A2S'} < \overline{N}^{A1S'}$, which implies that the most informative equilibrium under d' is still A-1-S. Applying part (ii) of Lemma 7, we have $E(U_p^{A1S'}) < E(U_p^{A1S})$.

Part (iii). The proof or construction of both cases is similar to that of part (ii), and thus is omitted. ■

Proposition 5 implies that in the most informative equilibrium making two agents' biases more unequal does not always improve or reduce the DM's expected payoff: sometimes it is better for two agents to have relatively equal biases and sometimes it is the opposite. In the following figure ($b_1 + b_2 = 0.196$), as d increases from 0 to 0.043, the most informative equilibrium is an A-2-S equilibrium with even number of partitions, and the DM's payoff first increases then decreases. For d bigger than 0.043, the most informative equilibrium is an A-1-S equilibrium with odd number of partitions, and the DM's payoff increases with d.

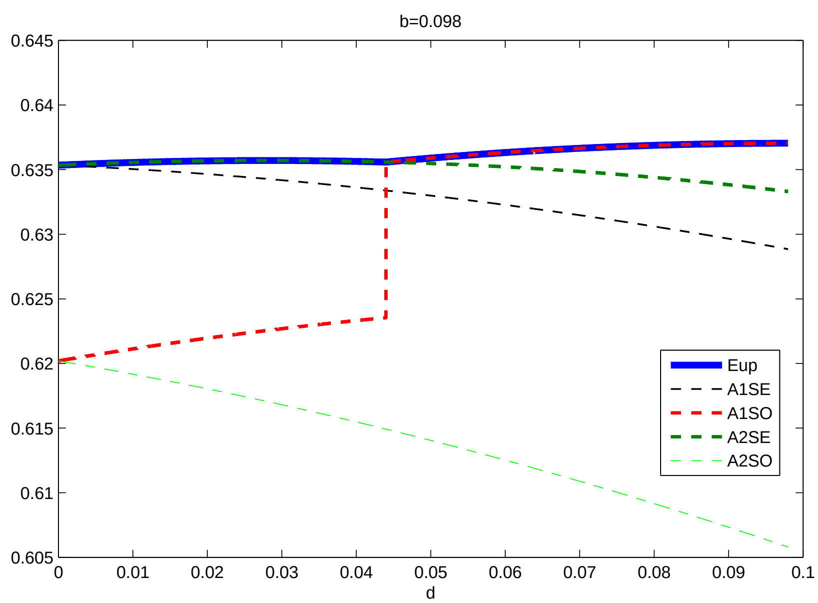

Figure 6: DM's Payoff as Biases Become More Unequal

Although in most cases the DM's expected payoff in the most informative equilibrium decreases with a larger total bias, it is possible that the DM's expected payoff could increase as the total bias increases, if the distribution of biases change as well. This is illustrated in the following example.

Example 5 . *Suppose $b_1 = 0.151$ and $b_2 = 0.175$. The most informative equilibrium is the A1SO equilibrium with 3 partitions (the A2SO equilibrium with 3 partitions does not exist) , and $E(U_p) = 0.6118$. Suppose $b_1 = 0.154$ and $b_2 = 0.173$. Note that, compared to the former case, b_1 increases, b_2 decreases, and the total bias increases. The most informative equilibrium is the A2SO equilibrium with 3 partitions (the A1SO equilibrium with 3 partitions still exists), and $E(U_p) = 0.6131$. That is, the DM's expected payoff increases.*

5 Sequential Communication and Delegation

5.1 Sequential communication

Now we consider the situation in which two agents communicate sequentially to the DM. In the first stage, one agent sends a message, which is publicly observable. Then, in the second stage, the other agent sends a message. Finally, the DM decides which project to implement. Note that there are two possible arrangements: either agent 1 sends message first or agent 2 sends message first.

Denote agent i as the agent who moves first and agent j as the agent who moves second. A strategy for agent i specifies a message m_i for each θ_i, which is denoted as the communication rule $\mu_i(m_i|\theta_i)$. A strategy for agent j specifies a message m_j for each pair of θ_j and m_i, which is denoted $\mu_j(m_j|\theta_j, m_i)$. A strategy for the DM specifies an action d for each message pair (m_i, m_j), which is denoted as decision rule $d(m_i, m_j)$. The DM's posterior beliefs on θ_i and θ_j after hearing messages are denoted as belief functions $g_i(\theta_i|m_i)$ and $g_j(\theta_j|m_j, m_i)$.

A Perfect Bayesian Equilibrium (PBE) requires:

(i) Given the DM's decision rule $d(m_1, m_2)$ and agent j's communication rule $\mu_j(m_j|\theta_j, m_i)$, agent i's communication rule $\mu_i(m_i|\theta_i)$ is optimal.

(ii) Given the DM's decision rule $d(m_1, m_2)$, agent i's communication rule $\mu_i(m_i|\theta_i)$, and agent i message m_i, agent j's communication rule $\mu_j(m_j|\theta_j, m_i)$ is optimal.

(iii) The DM's decision rule $d(m_1, m_2)$ is optimal given beliefs $g_i(\theta_i|m_i)$ and $g_j(\theta_j|m_j, m_i)$.

(iv) The belief functions $g_i(\theta_i|m_i)$ and $g_j(\theta_j|m_j, m_i)$ are derived from the agents' communication rules $\mu_i(m_i|\theta_i)$ and $g_j(\theta_j|m_j, m_i)$ according to Bayes rule whenever possible.

Denote $\overline{m}_i$ as the posterior induced by agent i's message m_i: $\overline{m}_i \equiv E[\theta_i|m_i]$.

Lemma 8 *In PBE the following properties hold. (i) Given any message of agent i, m_i, agent j, who moves second, has at most two irreducible messages: message h and l. (ii) Under message h, agent j's project is implemented with probability 1 generically; and under message l agent i's project is implemented for sure. When $\overline{m}_i \in (b_j, 1 - b_j)$, agent j could have two messages, and agent j sends message h if $\theta_j < \overline{m}_i - b_j$ and sends message l if $\theta_j > \overline{m}_i - b_j$. When $\overline{m}_i < b_j$, agent j essentially only has one message. When $\overline{m}_i > 1 - b_j$, agent j essentially only has one message, which is l. (iii) Agent i, who moves first, has an equilibrium strategy of interval form.*

Proof. Part (i). It is enough to rule out the case that agent j has three irreducible messages for some m_i, since the argument to rule out more than three irreducible messages is similar. Suppose, given m_i, agent j has three irreducible messages: l, m, and h. Let the probability that project j is implemented given m_j, $j = l, m, h$, be p_j. Since the messages are irreducible, these probabilities must be different. Without loss of generality, suppose $p_l < p_m < p_h$. It follows that $p_m \in (0,1)$. Now consider agent j's incentive. For all types of $\theta_j > \overline{m}_i - b_j$, agent j strictly prefers sending message h; for all types of $\theta_j < \overline{m}_i - b_j$ (this set might be empty), agent j strictly prefers sending message l; for type $\theta_j = \overline{m}_i - b_j$, agent j is indifferent among all three messages. Thus, message m can only be sent by the type of $\overline{m}_i - b_j$ of agent j. But, then from the DM's point of view, after hearing message m from agent j he should implement project i with probability 1. This contradicts the presumption that $p_m \in (0,1)$. Therefore, agent j can have at most two messages for any given m_i.

Part (ii). When $\overline{m}_i \in (b_j, 1-b_j)$, agent j could have two messages: h and l. If $\theta_j < \overline{m}_i - b_j$, then agent j sends message l and project i is implemented for sure, since in this case the posterior of θ_j is smaller than $\overline{m}_i$. If $\theta_j > \overline{m}_i - b_j$, then agent j sends message h and project j is implemented for sure. This is because the posterior of θ_j given message h is $(1+\overline{m}_i - b_j)/2$, which is greater than $\overline{m}_i$ since $\overline{m}_i < 1-b_j$. When $\overline{m}_i < b_j$, agent j essentially only has one message, as all types of θ_j want to send the same message. Depending on whether $\overline{m}_i$ is greater or less than 1/2, project j is implemented with probability 0 in the first case and with probability 1 in the second case. When $\overline{m}_i > 1-b_j$, agent j essentially only has one message, which is l. To see this, note that the types of $\theta_j \in (1-b_j, 1]$ will send message h, but the posterior of θ_j given message h, $(1+\overline{m}_i - b_j)/2$, is strictly less than $\overline{m}_i$ since $\overline{m}_i > 1-b_j$. Therefore, sending message h again leads to project i being implemented, which is essentially the same as sending message l.

Part (iii). Let $\mu_j(\cdot)$ be the equilibrium communication rule for agent j specified in part (ii). Suppose the realized state i is θ_i and agent i induces a posterior belief v_i of θ_i. Given the DM's optimal decision, agent i's expected utility can be written as

$$\begin{aligned} E_{\theta_j}[U_i|\theta_i, v_i] &= \Pr(\theta_j \le v_i - b_j)(\theta_i + b_i) + \Pr(\theta_j > v_i - b_j)E[\theta_j|\theta_j > v_i - b_j)] \\ &= v_i(\theta_i + b_i) + \frac{1}{2}[1-(v_i - b_j)^2] - b_j(\theta_i + b_i). \end{aligned}$$

From the above expression, it can be readily seen that $\frac{\partial^2}{\partial\theta_i\partial v_i}E_{\theta_j}[U_i|\theta_i, v_i] > 0$. This means that for any two different posterior of θ_i, say $\underline{v}_i < \overline{v}_i$, there is at most one type of agent i who is indifferent between $\underline{v}_i$ and $\overline{v}_i$. Therefore, agent i's equilibrium strategy must be of interval form. ■

Given the equilibrium strategies specified in Lemma 8, now we characterize equilibrium in more detail. We start with the case that agent 1 moves first.

5.1.1 Agent 1 talks first

Let N be the number of partitions of agent 1 and $a = (a_0, a_1, a_2, ..., a_N)$ be agent 1's partition points. Recall that $\overline{m}_n$ is the posterior given message m^n. In particular, $\overline{m}_n = (a_{n-1} + a_n)/2$. Note that $\overline{m}_1 \le 1/2$ for any N. Later on we will show that, for all n, $\overline{m}_n < 1-b_2$, and for all $n > 1$, $\overline{m}_n > b_2$. Thus the equilibrium strategies of agent 2 in part (ii) of Lemma 8 can be simplified further as follows. For all $n \ge 2$, given agent 1 sends message m^n, agent 2 has two messages h and l, and project 1 is implemented for sure after message l and project 2 is implemented for sure after message h. When agent 1 sends message m^1, agent 2 can either have one message or two messages, for which we discuss below.

Case 1: agent 2 has two messages when agent 1 sends message m^1. In this case, when agent 1 sends message m^1, agent 2 also has two messages, h and l, and project 1 is implemented for sure after message l and project 2 is implemented for sure after message h. We label this type of equilibrium as A-1-F-Case-1 equilibrium. Note that for agent 2 to have two messages after m^1, it must be the case that $\overline{m}_1 > b_2$ or $a_1 > 2b_2$. If $\theta_1 = a_n$, $1 \leq n \leq N-1$, agent 1 should be indifferent between sending m^n and m^{n+1}, which yields the indifference condition below

$$\begin{aligned} & (1-\overline{m}_n+b_2)\frac{1+\overline{m}_n-b_2}{2}+(\overline{m}_n-b_2)(a_n+b_1) \\ = \quad & (1-\overline{m}_{n+1}+b_2)\frac{1+\overline{m}_{n+1}-b_2}{2}+(\overline{m}_{n+1}-b_2)(a_n+b_1). \end{aligned} \tag{22}$$

The equation of (22) can be rearranged as,

$$(\overline{m}_{n+1}-\overline{m}_n)[\overline{m}_{n+1}+\overline{m}_n-2(b_1+b_2+a_n)]=0.$$

The above equation has two solutions: either $\overline{m}_{n+1}-\overline{m}_n=0$ or the term in the bracket equals to 0. Note that $\overline{m}_{n+1}-\overline{m}_n=0$ implies that there is a fully-revealing equilibrium for agent 1. We argue that this is impossible for the following reason. Suppose agent 1 fully reveals his information. Now consider any type of agent 1 with $\theta_1 \in (b_2, 1-b_2)$. Note that by Lemma 8 agent 2 has two messages, and he will send message h for $\theta_2 \geq \theta_1 - b_2$, in which case project 2 is implemented, and he will send message l for $\theta_2 < \theta_1 - b_2$, in which case project 1 is implemented. It is obvious that type θ_1 of agent 1 could increase his payoff by deviating to reporting as type $\theta_1+b_1+b_2$. Therefore, fully revealing equilibrium does not exist.

By ruling out $m_{n+1} \neq m_n$, we can further simplify the indifference condition (22) as

$$(a_{n+1}-a_n)-(a_n-a_{n-1})=4(b_1+b_2). \tag{23}$$

Equation (23) indicates that the partition size depends on both biases, and the incremental step size is $4(b_1+b_2)$, the same as that of asymmetric equilibrium under simultaneous communication. The reason for the incremental step size being $4(b_1+b_2)$ is as follows. Agent 1 anticipates that agent 2 will exaggerate his state by b_2 (the indifference type of agent 2 between sending messages h and l is $\theta_1 - b_2$). Combining with agent 1's own bias b_1, for type θ_1 agent 1 ideally would report as type $\theta_1+b_1+b_2$, resulting in agent 2's indifferent type being his own ideal cutoff θ_1+b_1. Thus agent 1's effective bias becomes b_1+b_2, which leads to the incremental step size being $4(b_1+b_2)$. The difference equation can be solved as

$$a_n = \frac{n}{N} - 2(b_1+b_2)n(N-n).$$

And the necessary condition for an equilibrium with N partitions is

$$2(b_1+b_2)(N-1)N < 1, \tag{24}$$

which gives the upper bound of number of partitions N, $\overline{N}^{1F}_{C1} = \langle \frac{1}{2} + \frac{1}{2}(1 + \frac{2}{b_1+b_2})^{1/2} \rangle$.

Now let us go back to check whether, for all n, $\overline{m}_n < 1 - b_2$ holds. We only need to show that it holds for the maximum $\overline{m}_n$, $\overline{m}_N$. Specifically, for $N = 1$, $\overline{m}_N = 1/2 < 1 - b_2$ since $b_2 < 1/2$. For $N \geq 2$, $1 - m_N > 4(b_1 + b_2)$, which implies that $\overline{m}_N < 1 - 2(b_1 + b_2) < 1 - b_1$. Similarly, the difference equation (23) implies that $\overline{m}_n > b_2$ for all $n > 1$.

Finally, the constraint that $a_1 \geq 2b_2$ can be explicitly written as

$$1 - 2b_2N^2 - 2b_1N(N-1) \geq 0. \tag{25}$$

Case 2: agent 2 has only one message when agent 1 sends message m^1. In this case, when agent 1 sends message m^1, agent 2 only has one message. For $N \geq 2$, we have $\overline{m}_1 < 1/2$, so project 2 will be implemented for sure (or agent 2's single message is h). We label this type of equilibrium as A-1-F-Case-2 equilibrium. One may wonder that, for this case to arise, we must have $\overline{m}_1 \leq b_2$ or $a_1 \leq 2b_2$. In other words, it is impossible for agent 2 to send two messages after m^1. But it turns out that the condition $\overline{m}_1 \leq b_2$ is not necessary. To see this, suppose $\overline{m}_1 > b_2$ or $a_1 > 2b_2$. Consider the following strategies after agent 1 sending message m^1: the DM implements project 2 regardless of agent 2's message, and agent 2 sends only one message. This is clearly a part of equilibrium. Given that the DM ignores agent 2's message, agent 2 essentially only has one message. And, given that agent 2 has only one message, the posterior of θ_2 is $1/2 \geq \overline{m}_1$; thus it is optimal for the DM to implement project 2.

It can be verified that the indifference condition for $n \geq 2$ is the same as that in case 1: $(a_{n+1} - a_n) - (a_n - a_{n-1}) = 4(b_1 + b_2)$. For partition point a_1, the indifference condition becomes

$$\begin{aligned} \frac{1}{2} &= (1 - \overline{m}_2 + b_2)\frac{1 + \overline{m}_2 - b_2}{2} + (\overline{m}_2 - b_2)(a_n + b_1) \\ \Leftrightarrow &\quad a_2 - a_1 = 2a_1 + 2b_2 + 4b_1. \end{aligned} \tag{26}$$

Solving the difference equations, we get

$$a_1 = \frac{1}{2N-1}[1 - 2b_2(N-1)^2 - 2b_1N(N-1)].$$

The necessary condition for an equilibrium with N partitions is

$$2b_2(N-1)^2 + 2b_1N(N-1) < 1. \tag{27}$$

Denote $\overline{N}^{1F}_{C2}$ as the the upper bound of the number of partitions.

5.1.2 Agent 2 talks first

The analysis for the case that agent 2 talks first is very similar to that of agent 1 talks first (only the roles of b_1 and b_2 are reversed), and we just report the results.

Case 1: agent 1 has two messages when agent 2 sends message m^1. We label this type of equilibrium as A-2-F-Case-1 equilibrium. Note that for agent 1 to have two messages after m^1, it must be the case that $\overline{m}_1 > b_1$ or $a_1 > 2b_1$. Let $\overline{N}_{C1}^{2F}$ be the maximum equilibrium number of partitions. The equilibrium characterization is parallel to that in the case that agent 1 talks first. The only difference is that, constraint (25) is replaced by

$$1 - 2b_1 N^2 - 2b_2 N(N-1) \geq 0. \tag{28}$$

Case 2: agent 1 has only one message when agent 2 sends message m^1. In this case, when agent 2 sends message m^1, agent 1 only has one message, and project 2 will be implemented for sure. We label this type of equilibrium as A-2-F-Case-2 equilibrium. The necessary condition for a partition equilibrium with N is

$$2b_1(N-1)^2 + 2b_2 N(N-1) < 1. \tag{29}$$

Denote $\overline{N}_{C2}^{2F}$ as the the upper bound of the equilibrium number of partitions.

5.1.3 Comparison to simultaneous talk

The following proposition shows that the equilibria under sequential talk and those under simultaneous talk are equivalent.

Proposition 6 *If there is an equilibrium under simultaneous talk, then there is a corresponding equilibrium under sequential talk which implements the same outcome; and vice versa. Specifically, the correspondence of equilibria is as follows: (i) A-1-S equilibria with even number of partitions under simultaneous talk are equivalent to A-1-F-Case-2 equilibria under sequential talk; (ii) A-1-S equilibria with odd number of partitions under simultaneous talk are equivalent to A-2-F-Case-1 equilibria under sequential talk; (iii) A-2-S equilibria with even number of partitions under simultaneous talk are equivalent to A-2-F-Case-2 equilibria under sequential talk; (iv) A-2-S equilibria with odd number of partitions under simultaneous talk are equivalent to A-1-F-Case-1 equilibria under sequential talk.*

The difference between simultaneous talk and sequential talk is that, under sequential talk the agent who talks the second can condition his message on the first agent's message, and thus only has at most two messages condition on the first agent's message. But if we recover the unconditional messages of the second agent, then equilibrium under sequential talk and equilibrium under simultaneous talk become directly comparable.

Example 6 *The following figure illustrates an A-1-F-Case-2 equilibrium under sequential talk that is equivalent to the A-1-S equilibrium under simultaneous talk. The dotted line indicates the posterior of θ_1 given agent 1's messages. When agent 1 sends the highest message, agent 2's cutoff is a_{22} and he sends a high message if and only if $\theta_2 \geq a_{22}$. When agent 1 sends the second highest* *message, agent 2's cutoff is a_{21}. In total, agent 2 has three unconditional messages (partitions). When agent 1 sends the highest message, agent 2's two lower unconditional messages are combined to the low conditional message. When agent 1 sends the second highest message, agent 2's two higher unconditional messages are combined to the high conditional message.*

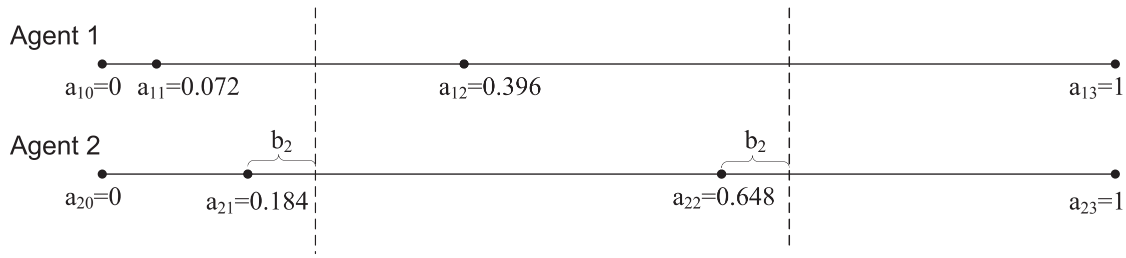

Figure 7: The Equivalence of A-1-S and A-1-F-Case-2 Equilibria

The equivalence between simultaneous talk and sequential talk under competitive cheap talk is new and surprising. In other cheap talk models with multiple senders, simultaneous talk and sequential talk usually lead to different outcomes.[17]

Why the second agent's ability, under sequential talk, to condition his message on the first agent's message does not change the equilibrium outcome? The underlying reason is that, since only the comparison of two projects matters, even under simultaneous talk one agent's message matters only if the ranking of his message is adjacent to that of the other agent's message. In other words, when one agent decides which message to send after observing his own state, he has already implicitly conditioned on that the other agent's message has adjacent rankings. This implies that, under sequential talk, the second agent's ability to directly condition his message on the first agent's message does not matter.

Combining the equivalence results of Proposition 6 and the results of Proposition 3 regarding the most informative asymmetric equilibrium under simultaneous talk, we can characterize the most informative equilibrium under sequential talk in a straightforward way. Although we will not elaborate on the characterization, a general pattern is that in the the most informative equilibrium under sequential talk either agent could talk first. Therefore, from the DM's perspective, letting who talk first depends on specific situations. But in the most informative equilibrium, it is more likely that the agent having a bigger bias talks first (arise in two out of three scenarios).[18]

[17]For example, in Krishna and Morgan (2001b) where two agents have symmetirc opposing biases and communicate simultaneously (corresponding to open rules with heterogenous committee), full information revelation is achievable in equilibrium. However, in Krishna and Morgan (2001b) where two agents have opposing biases and communicate sequenitally, full information revelation is not achievable. In a model in which agents' biases are private information, Li (2010) shows that sequential talk is superior to simultaneous communication.

[18]In this respect, our paper is related to Ottaviani and Sorensen (2001), who study the order of public speech with agents having different abilities and reputational concerns.

Proposition 7 *Suppose agent i talks first under sequential communication. In any equilibrium, (i) the ex ante probability that agent j's project being implemented is strictly greater than* $1/2$. *(ii) Agent j' ex ante payoff is higher than that of agent i.*

Proof. Part (i). By Proposition 6, A-i-F-Case-2 equilibria under sequential talk are equivalent to A-i-S equilibria with even number of partitions under simultaneous talk. Now by part (ii) of Proposition 4, the ex ante probability that agent i's (j's) project being implemented is strictly less (greater) than 1/2. Similarly, by Proposition 6, A-i-F-Case-1 equilibria under sequential talk are equivalent to A-j-S equilibria with odd number of partitions under simultaneous talk. Now by part (i) of Proposition 4, the ex ante probability that agent j's (i's) project being implemented is strictly greater (less) than 1/2.

Part (ii). Since in any equilibrium both agents get the same return from the implemented project, the result immediately follows part (i). ■

Proposition 7 indicates that the agent who talks first is always worse off compared to the agent who talks the second. This is because in terms of unconditional messages the agent who talks the second under sequential talk always has the sure option (or the highest overall message) under simultaneous talk. However, this does not mean that each agent prefers to be the one who talks the second. This is because, when the order of communication is changed, the most informative equilibrium changes as well. So the agent who talks first might still prefer talking first, as he anticipates that switching the order of talking might lead to a less informative equilibrium, under which he is worse off.

5.2 Delegation

We only consider the case of simple delegation (Aghion and Tirole, 1997; Dessein, 2002).[19] That is, the DM delegates the decision right to one of the agents, say agent i. Since agent i cares about the quality of the project implemented, he first consults agent j regarding θ_j, and then makes the decision as to which project to implement. In this setting of simple delegation, there are two possibilities: the decision right is delegated to either agent 1 or agent 2. We call the former case A-1 delegation and the latter A-2 delegation.

Consider A-1 delegation, in which agent 2 sends messages first and then agent 1 makes the final decision. Given the posterior $\overline{m}_n$ induced by agent 2's message m^n regarding θ_2, agent 1's optimal decision is easy to characterize: implemented project 1 if $\theta_1 + b_1 > \overline{m}_n$ and implement project 2 otherwise. As to agent 2, we can show that (similar to part (iii) of Lemma 8 under sequential talk) his equilibrium strategy is of interval form.

Proposition 8 *(i) The set of equilibria under simple delegation is a subset of that under sequential talk. Specifically, for any A-i-F-Case-1 equilibrium under sequential talk, there is a corresponding equilibrium under A-j delegation. For any A-i-F-Case-2 equilibrium under sequential talk, there is a corresponding equilibrium under A-j delegation if and only if $a_{1,i} \le 2b_j$. (ii) The most informative equilibrium under sequential talk and that under simple delegation is equivalent.*

[19]For optimal delegation, see Melumad and Shibano (1991) and Alonso and Matouschek (2008).

Proof. We only need to prove the relationship between A-2-F equilibrium under sequential talk and equilibrium under A-1 delegation, since the relationship between A-1-F equilibrium under sequential talk and equilibrium under A-2 delegation is similar.

Part (i). Consider sequential talk with agent 2 talks first (A-2-F). According to previous analysis, all equilibria have the following feature. The DM's final decision always follows agent 1's message: if agent 1 sends a high message, then agent 1's project is implemented, and project 2 is implemented if agent 1 sends a low message. Thus, it is as if agent 1 has the decision rights.

As to agent 1's equilibrium strategy, we discuss two cases. First, consider Case 1 equilibria under sequential talk. Note that agent 1's equilibrium strategy under talk is the same as his equilibrium strategy under delegation: sends a high message (implement project 1) if and only if $\theta_1 + b_1 > \overline{m}_n$. Given this, agent 2's equilibrium strategy must be also the same under two scenarios. Therefore, for any A-2-F-Case-1 equilibrium under sequential talk, there is a corresponding equilibrium under A-1 delegation.

Next, consider Case 2 equilibria under sequential talk. The equivalence between equilibria under two scenarios is similar to the previous case, except for one difference. Recall that under sequential talk when agent 2 sends the lowest message, agent 1 only has one message, which is message h, and project 1 is implemented for sure. Under A-1 delegation, agent 1's strategy is still the same as in the previous case: implement project 1 if and only if $\theta_1 + b_1 > \overline{m}_n$. To make sure that project 1 is implemented for sure when agent 2 sends the lowest message, an additional constraint $a_{1,2} \leq 2b_1$ must be satisfied. But this condition is not required under sequential talk. Therefore, for any A-2-F-Case-2 equilibrium under sequential talk, there is a corresponding equilibrium under A-1 delegation if and only if $a_{1,2} \leq 2b_1$. Combining the two cases, we reach the conclusion that the set of equilibria under simple delegation is a subset of that under sequential talk.

Part (ii). Given the results in part (i), it is sufficient to show that if there is a A-2-F-Case 2 equilibrium with agent 2 having N partitions, and $a_{1,2} > 2b_1$, then it cannot be the most informative equilibrium. It is enough to find another equilibrium which is more informative than the one mentioned. The condition $a_{1,2} > 2b_1$ can be written more explicitly as

$$1 - 2b_1 N^2 - 2b_2 N(N-1) > 0.$$

This implies that

$$2(b_1 + b_2)(N-1)N < 1.$$

The above two inequalities means that the A-2-F-Case 1 equilibrium with agent 2 having N partitions exists. But this equilibrium is more informative than the one mentioned earlier. This is because, in terms of corresponding asymmetric equilibria, the former equilibrium is an A-1-S equilibrium with $2N+1$ total number of partitions, while the latter equilibrium is an A-2-S equilibrium with $2N$ total number of partitions. ■

The intuition for Proposition 8 is as follows. In any equilibrium under sequential talk, the DM's decision always follows the suggestion (message) of the second agent.[20] Thus under sequential talk it is as if the second agent has the decision right, which is equivalent to simple delegation.

[20] Recall that the biases of both agents are smaller than $1/2$. This implies that the agent who talks second can always transmit some information.

The only difference is that with simple delegation, the agent who has decision right will always utilize his own private information, while under sequential talk the second agent's message might be ignored. Therefore, the set of equilibria under simple delegation is a subset of that under sequential talk. However, sequential talk and simple delegation always lead to the same most informative equilibrium; thus they are essentially equivalent. Why, under sequential talk, the DM's decision always follows the suggestion (message) of the second agent? Roughly speaking, given the first agent's equilibrium messages or partitions, the amount of private information possessed by the second agent always outweighs his incentive to exaggerate the return of his own project. This is because when the first agent determines his partitions he already taken into account the second agent's bias. In particular, each partition size of the first agent is always bigger than twice of the second agent's bias.

Combining with previous results, we conclude that simultaneous talk, sequential talk, and simple delegation are essentially all equivalent, in terms of the most informative equilibrium. This result is quite surprising, as in other cheap talk models cheap talk and simple delegation in general lead to different equilibrium outcomes.[21] Moreover, the agent having the decision rights is always better off relative to the other agent.

6 More Than Two Agents

Now we go back to the setting of simultaneous communication, and study the situation where there are more than two agents.

6.1 Symmetric agents

Suppose there are $k \geq 2$ agents and all agents have the same bias b. All the other assumptions are the same as in the basic model. By modifying the proof of Proposition 1 slightly, we can show that all PBE must be interval equilibria. We are interested in symmetric equilibria in which all agents play the same strategy (have the same partitions) and the DM treat all agents equally: in case that m agents tie for the highest message, the DM implements each of those agents' projects with the same probability $1/m$. To characterize the equilibrium partition points a, suppose agent i's realized return $\theta_i = a_n$. He should be indifferent between sending messages m^n and m^{n+1}, which gives rise to the following equation:

$$\begin{aligned} & [\frac{a_{n+1}+a_n}{2} - (a_n + b)][\sum_{m=1}^{k-1} C_{k-1}^m (a_{n+1} - a_n)^m a_n^{k-1-m} \frac{1}{m+1}] \\ = \quad & [(a_n + b) - \frac{a_n + a_{n-1}}{2}][\sum_{m=1}^{k-1} C_{k-1}^m (a_n - a_{n-1})^m a_{n-1}^{k-1-m} \frac{m}{m+1}]. \end{aligned} \tag{30}$$

[21] For instance, Dessein (2002) shows that simple delegation is strictly better than cheap talk whenever informative cheap talk is feasible. In a two-sender model, which is more comparable to the current model, McGee and Yang (2013) shows that simple delegation is strictly better than simultaneous talk if two agents have like biases, and it can be better or worse than simultaneous talk if two agents have opposing biases.

The LHS of (30) is the expected loss, and the RHS of (30) is the expected gain, of sending the higher message m^{n+1}.

Inspecting (30), we can see that for $k > 2$ the difference equation is highly nonlinear, which means that explicit solution is infeasible. Nevertheless, we will show that with more agents, the incremental step size of partitions will be smaller. We proceed with two lemmas.

Lemma 9 *The following two equations hold:*

$$\begin{aligned}\sum_{m=1}^{k} C_k^m (b-a)^{m-1} a^{k-m} \frac{1}{m+1} &= \frac{-a^k b + b^{k+1} + a^{k+1}k - a^k bk}{(a-b)^2(1+k)},\\ \sum_{m=1}^{k} C_k^m (a-b)^{m-1} b^{k-m} \frac{m}{m+1} &= \frac{-a^k b + b^{k+1} + a^{k+1}k - a^k bk}{(a-b)^2(1+k)}.\end{aligned}$$

Lemma 10 *Suppose* $0 < c < a < b < 1$, *and* $k \in Z^+, k > 1$. *Then, the following inequality holds:*

$$\frac{\sum_{m=1}^{k-1} C_{k-1}^m (b-a)^{m-1} a^{k-1-m} \frac{1}{m+1}}{\sum_{m=1}^{k-1} C_{k-1}^m (a-c)^{m-1} c^{k-1-m} \frac{m}{m+1}} < \frac{\sum_{m=1}^{k} C_k^m (b-a)^{m-1} a^{k-m} \frac{1}{m+1}}{\sum_{m=1}^{k} C_k^m (a-c)^{m-1} c^{k-m} \frac{m}{m+1}}.$$

Proposition 9 *As the number of agents, k, increases, in symmetric equilibrium the incremental step size of partitions decreases.*

Proof. The indifference condition (30) can be reformulated as

$$\begin{aligned}&\frac{a_{n+1} - a_n - 2b}{2}(a_{n+1} - a_n)\frac{\sum_{m=1}^{k-1} C_{k-1}^m (a_{n+1} - a_n)^m a_n^{k-1-m} \frac{1}{m+1}}{\sum_{m=1}^{k-1} C_{k-1}^m (a_n - a_{n-1})^m a_{n-1}^{k-1-m} \frac{m}{m+1}}\\ &= \frac{a_n - a_{n-1} + 2b}{2}(a_n - a_{n-1})\end{aligned} \tag{31}$$

With $k' = k + 1$ senders, the indifference condition becomes

$$\begin{aligned}&\frac{a'_{n+1} - a'_n - 2b}{2}(a'_{n+1} - a'_n)\frac{\sum_{m=1}^{k} C_k^m (a'_{n+1} - a'_n)^m a'^{k-m}_n \frac{1}{m+1}}{\sum_{m=1}^{k} C_k^m (a'_n - a'_{n-1})^m a'^{k-m}_{n-1} \frac{m}{m+1}}\\ &= \frac{a'_n - a'_{n-1} + 2b}{2}(a'_n - a'_{n-1})\end{aligned} \tag{32}$$

Suppose $a'_j = a_j$, $j = n-1, n, n+1$. By Lemma 10 ($b = a_{n+1}$, $a = a_n$ and $c = a_{n-1}$), we have

$$\frac{\sum_{m=1}^{k-1} C_{k-1}^m (a_{n+1} - a_n)^{m-1} a_n^{k-1-m} \frac{1}{m+1}}{\sum_{m=1}^{k-1} C_{k-1}^m (a_n - a_{n-1})^{m-1} a_{n-1}^{k-1-m} \frac{m}{m+1}} < \frac{\sum_{m=1}^{k} C_k^m (a'_{n+1} - a'_n)^{m-1} a'^{k-m}_n \frac{1}{m+1}}{\sum_{m=1}^{k} C_k^m (a'_n - a'_{n-1})^{m-1} a'^{k-m}_{n-1} \frac{m}{m+1}}. \tag{33}$$

Now by (31) and (33), the LHS of (32) is strictly bigger than its RHS. This implies that, if $a'_{n-1} = a_{n-1}$ and $a'_n = a_n$, then $a'_{n+1} < a_{n+1}$. In other words, the incremental step size decreases as k increases. ■

The underlying reason for Proposition 9 is as follows. Recall that the indifference condition balances the expected gain (when no one among the other agents sends the higher message) and the expected loss (when some agents send the higher message) of sending the higher message. When the number of agents increases, other things equal, if the agent in question sends the higher message, relative to the probability of gaining (no one among the other agents sends the higher message), the probability of incurring loss (some agents send the higher message) increases. As a result, the agent in question has a weaker incentive to send the higher message or exaggerate the return of his own project, meaning that the partition size of the higher message becomes smaller relative to the partition size of the lower message.

Proposition 9 implies that with more agents the partition sizes will increase more slowly. Therefore, with more agents the maximum number of partitions in equilibrium will weakly increase, and equilibrium partitions will become more even. In other words, as the number of agents increases each agent will transmit more information in symmetric equilibrium. This pattern is illustrated in the following example.

Example 7 *Suppose $b = 0.4$. When there are two agents, the most informative symmetric equilibrium has two partitions, with partition point $a_1 = 0.1$. When there are three agents, in the two-partition equilibrium the partition point is $a_1 = 0.1572$. Clearly, when there are three agents, the incremental step size is smaller and the partitions are more even, and hence more information is transmitted by each agent.*

6.2 Asymmetric agents

Here we just discuss the case of three asymmetric agents, with $0 < b_1 < b_2 < b_3$. All PBE still must be interval equilibrium, and equilibrium must be asymmetric. All equilibrium messages of all three agents can be ranked unambiguously according to the posteriors.[22] To make the set of messages irreducible, two messages having the consecutive overall rankings must belong to different agents. However, with three agents the messages do not need to have an alternating ranking structure (unlike in the two-agent case): the overall rankings of three agents' messages have a cyclical pattern (for example, the lowest message belongs to agent 1, the second lowest to agent 2, the third lowest to agent 3, the fourth lowest to agent 1, and so on). There are many other possibilities, as long as two messages having the consecutive overall rankings belong to different agents. For example, agent 3 babbles (only has one message), and the messages of agent 1 and agent 2 basically have an alternating ranking structure (excluding agent 3's sole message). Essentially, only agent 1 and 2 are actively competing with each other, with agent 3's project (with expected payoff 1/2) serving as an outside option. Alternatively, one can think of complicated ranking structures in which three agents' messages or partitions are intertwined. There are a few interesting questions to ask. What kind of ranking structure will emerge in the most informative equilibrium? Is it better to have only two agents competing actively or to have all three agents

[22] With three agents actively competing with each other, Quasi-symmetric equilibrium defined in the two-agent case no longer exists. This is because now it is impossible to make two agents indifferent at the same time by manipulating the tie-breaking rule.

competing actively? To maximize the DM's payoff, should the agent who has the smallest bias always have the give-up option, or the sure option? We leave this topic for future research.

7 Conclusions and Discussions

This paper studies a competitive cheap talk model in which two agents, who each is responsible for a single project, communicate with the DM before exactly one project is chosen. Both agents and the DM share some common interests, but at the same time each agent has an own project bias. We first fully characterize the equilibria under simultaneous communication. All equilibria are shown to be partition equilibrium, and the partitions of two agents' are intimately related: the interior partition points of two agents has an alternating structure. The equilibrium numbers of partitions of two agents are either the same or differ by one. Although letting the agent with the smaller bias have the give-up option potentially leads to more partitions, in the most informative equilibrium the agent who has the bigger bias could have the give-up option. However, in the most informative equilibrium, the agent with a smaller bias has weakly more messages, and is more likely to have the give-up option and the sure option. Fixing the total bias of two agents, making the biases more unequal could increase or decrease the DM's payoff in the most informative equilibrium.

We then study sequential communication. It turns out that the set of equilibria under sequential communication and that under simultaneous communication are outcome equivalent. We also show that sequential communication and simple delegation are essentially equivalent in the sense that they always lead to the same most informative equilibrium. These are surprising results, as in other cheap talk models different timing and delegation typically lead to different equilibrium outcomes. Comparing two agents' payoffs, under simultaneous communication the agent who has the sure option, under sequential communication the agent who talks the second, and under simple delegation the agent who has the decision rights, is always relatively better off. When the number of agents increases, in the most informative symmetric equilibrium each agent transmits more information.

Throughout the paper we have assumed that the return of each project is uniformly distributed. With more general distributions, the difference equations will not have analytical solutions, which would complicate the analysis. However, we think that majority of the results of our paper will hold qualitatively under more general distributions. In the paper we also assumed that exactly one project will be implemented. In some situations, it is reasonable to think that there is an outside option under which neither project is implemented. If the DM chooses the outside option, then neither agent gets private benefit. With the outside option, if a project's return is too low, then the agent would rather have the outside option implemented, and thus his interest is perfectly aligned with the DM's. Therefore, for each agent there is a lowest message which essentially indicates the return of the project is definitely below the outside option. Apart from the lowest message, if either agent sends higher messages then the DM will definitely not adopt the outside option. In other words, starting from the second lowest messages two agents are competing with each other to have his own project implemented, which is essentially the same as the basic model. From this discussion, we can see that adding an outside option would not qualitatively change the existing results much. Finally, it is also interesting to study the case in which two projects are asymmetric or their returns have different distributions. We leave this for future research.

Appendix

Equilibrium selection.

Consider the following equilibrium refinement. In an A-i-S equilibrium suppose $a_{i,1} > 2b_j$ (or $\overline{m}_{i,1} > b_j$). Note that in equilibrium, if agent i sends the lowest message m_i^1 then project j is implemented for sure. Now suppose the realized return of project j is very low: $\theta_j \in [0, a_{i,1} - 2b_j)$. In this case, both agent j and the DM would prefer project i being implemented, given agent i strategy. To achieve that, agent j could send an out of equilibrium message, say "θ_j is very low" or "do not implement my project j," and the DM would listen to it and implement project i. This shows that an A-i-S equilibrium with $a_{i,1} > 2b_j$ is not stable or reasonable; and for an A-i-S equilibrium to be stable it must be the case that $a_{i,1} \leq 2b_j$.

Lemma 11 *(i) Suppose an A-i-S equilibrium with an even (odd) number of partitions $N+1$ is not the most informative A-i-S equilibrium with even (odd) number of partitions, then the equilibrium with $N+1$ partitions is not stable. (ii) The most informative A-1-S equilibrium must be stable.*

Proof. Part (i). We only prove the case that $N+1$ is even, since the proof for the case that $N+1$ is odd is similar. Translating into the context of QSE, A-i-S QSE with N partitions and $N+2$ partitions both exist. We want to show that $a_{i,1}(N) > 2b_j$. Given that a QSE with $N+2$ partitions exists, by (18) we have $\frac{(N+1)(N+3)}{2}b_i + \frac{(N+1)^2}{2}b_j < 1$. More explicitly, by (14),

$$a_1(N) - 2b_j \propto 2 - (N-1)(N+1)b_i - (N+1)^2 b_j > 0,$$

where the inequality follows the previous one.

Part (ii). Let $\overline{N}$ be the number of partitions in the most informative A-1-S QSE. We only prove the case that $\overline{N}$ is odd. We need to show that $a_{1,1}(\overline{N}) \leq 2b_2$. Suppose to the contrary $a_{1,1}(\overline{N}) > 2b_2$. By (14), it implies that

$$2 - (\overline{N}-1)(\overline{N}+1)b_1 - (\overline{N}+1)^2 b_2 > 0.$$

But given that $b_2 > b_1$, the above inequality implies that

$$2 - (\overline{N}-1)(\overline{N}+1)b_2 - (\overline{N}+1)^2 b_1 > 0,$$

which, by (17), implies that an A-1-S QSE with $\overline{N}+1$ partitions exists. This contradicts the fact that the most informative A-1-S QSE has $\overline{N}$ partitions. ■

The results of Lemma 11 are intuitive. If an equilibrium with more partitions exists, it implies that the first partition in the equilibrium of fewer partitions is large relative to the biases, which further means that the equilibrium with fewer partitions is not stable. Although Lemma 11 does not establish that the most informative equilibrium must be stable and any equilibrium that is not the most informative one is not stable, it suggests that only the more informative equilibria can be potentially stable and thus are the more reasonable ones.

Proof of Proposition 6.

Proof. Since the proof of each part is similar, we will show part (i) in detail only. For other parts, we will just show what the differences are from part (i).

Part (i). We need to show the equivalence between an A-1-F-Case-2 equilibrium under sequential talk with agent 1 having N partitions and an A-1-S equilibrium with $2N$ (even number) partitions under simultaneous talk. We prove it in two steps. First, we show that two equilibria lead to the same partitions. Second, we show that two equilibria implement the same outcome.

Recall that we denote agent 1's equilibrium partition points as a_1. We first show the difference equations characterizing a_1 under sequential talk and those under simultaneous talk are the same. Comparing the first equation in (8) under simultaneous talk and (23), we can see that they are exactly the same. Under simultaneous talk, the third equation in (8) and $a_{2,1} = \frac{1}{2}(a_{1,1} + a_{1,2}) - b_2$ exactly give rise to (26) under sequential talk. Thus the difference equations characterizing a_1 are the same under two scenarios.

As to agent 2's partition points a_2, consider the A-1-F-Case-2 equilibrium under sequential talk. Recall that agent 2, who talks second, has only two messages (or one message) conditional on agent 1's message. Now we augment agent 2's messages (or recover agent 2's unconditional partitions). Denote $\overline{m}_{1,n}$ as the posterior of θ_1 induced by agent 1's message m_1^n. Now define agent 2's unconditional partitions as follows. For $N \geq n \geq 2$, define $a_{2,n-1} = \overline{m}_{1,n} - b_2$, and agent 2 sends message m_2^n if $\theta_2 \in [a_{2,n-1}, a_{2,n}]$. We denote the constructed partition points of agent 2 as a_2. It can be readily seen that agent 2 in total has N unconditional messages. By construction, note that for any interior n we have $a_{2,n} > a_{1,n}$. Thus (a_1, a_2) under simultaneous talk is potentially corresponding to Agent-1-sacrificing equilibria with an even number $(2N)$ of partitions under simultaneous talk.

Now we show that the constructed a_2 under sequential talk follows the same equilibrium indifference conditions under simultaneous talk. Actually, this is obvious by construction. For $N > n \geq 1$, we have $a_{2,n} = \overline{m}_{1,n+1} - b_2$, which is exactly the same as (7) under simultaneous talk. Therefore, two equilibria lead to the same partitions for both agents.

What remains to be shown is that the two equilibria implement the same outcome. Recall that we have already shown that after augmenting the messages (recovering the unconditional partitions) of agent 2 under sequential talk, two equilibria lead to the same partitions for both agents. Now consider any realized pair of states $(\theta_{1,n_1}, \theta_{2,n_2})$ (θ_1 belongs to partition n_1, and θ_2 belongs to partition n_2). Under simultaneous talk, project 1 is implemented if and only if $n_1 > n_2$, otherwise project 2 is implemented. Under sequential talk, if $n_1 > n_2$ then agent 2 will send a low message and project 1 is implemented; if $n_1 \leq n_2$ then agent 2 will send a high message and project 2 is implemented. This shows that the two equilibria implement the same outcome.

Part (ii). Consider an A-2-F-Case-1 equilibrium under sequential talk with agent 2 having N partitions. Since for each message of agent 2 agent 1 has two messages, in total agent 1 has $N + 1$ unconditional messages or partitions. Moreover, if agent 1 sends the lowest unconditional message, project 1 will not be implemented for sure. Therefore, it corresponds to A-1-S equilibria with an odd number of partitions ($2N+1$ partitions in total).

Part (iii). Consider an A-2-F-Case-2 equilibrium under sequential talk with agent 2 having N partitions. Since, for the lowest message of agent 2 agent 1 only has one message, and for each other message of agent 2 agent 1 has two messages, in total agent 1 has N unconditional messages or partitions. Moreover, if agent 2 sends the lowest message, project 2 will not be implemented for sure. Therefore, it corresponds to A-2-S equilibria with an even number of partitions ($2N$ partitions in total).

Part (iv). Consider an A-1-F-Case-1 equilibrium under sequential talk with agent 1 having N partitions. Since for each message of agent 1 agent 2 has two messages, in total agent 2 has $N+1$ unconditional messages or partitions. Moreover, if agent 2 sends the lowest unconditional message, project 2 will not be implemented for sure. Therefore, it corresponds to A-2-S equilibria with an odd number of partitions ($2N+1$ partitions in total). ■

Proof of Lemma 9.

Proof. We only prove the first statement, as the proof for the second statement is similar.

$$\begin{aligned}\sum_{m=1}^{k} C_k^m (b-a)^{m-1} a^{k-m} \frac{1}{m+1} &= \sum_{m=1}^{k}\sum_{i=0}^{m-1} C_k^m C_{m-1}^i b^i (-1)^{m-1-i} a^{k-i-1} \frac{1}{m+1} \\ &= \sum_{i=0}^{k-1} b^i a^{k-i-1} \Big(\sum_{m=i+1}^{k} C_k^m C_{m-1}^i (-1)^{m-1-i} \frac{1}{m+1}\Big) \\ &= \frac{-a^k b + b^{k+1} + a^{k+1}k - a^k bk}{(a-b)^2(1+k)}.\end{aligned} \tag{34}$$

■

Proof of Lemma 10.

Proof. By Lemma 9, the above inequality is equivalent to

$$\frac{-a^{k-1}b + b^k + a^k(k-1) - a^{k-1}b(k-1)}{-a^{k-1}c + c^k + a^k(k-1) - a^{k-1}c(k-1)} < \frac{-a^k b + b^{k+1} + a^{k+1}k - a^k bk}{-a^k c + c^{k+1} + a^{k+1}k - a^k ck}.$$

The above inequality can be further simplified as $(a^k - b^k)(b-c)(a^k - c^k) + (b-a)(a-c)(b^k - c^k)a^{k-1}k > 0$. Given that $c < a < b$, this inequality is equivalent to

$$-\Big(\sum_{i=0}^{k-1} b^{k-i-1}a^i\Big)\Big(\sum_{i=0}^{k-1} a^{k-i-1}c^i\Big) + \Big(\sum_{i=0}^{k-1} b^{k-i-1}c^i\Big)a^{k-1}k > 0. \tag{35}$$

We show that inequality (35) holds by induction. Let $A_k \equiv \sum_{i=0}^{k-1} b^{k-i-1}a^i$, $B_k \equiv \sum_{i=0}^{k-1} b^{k-i-1}c^i$ and $C_k \equiv \sum_{i=0}^{k-1} a^{k-i-1}c^i$. For $k=2$, inequality (35) becomes $(b-a)(a-c) > 0$, which obviously holds. Now suppose inequality (35) holds for $n=k$, that is, $A_k C_k < B_k a^{k-1}k$. We want to show inequality (35) holds for $n = k+1$, that is, $A_{k+1}C_{k+1} < B_{k+1}a^k(k+1)$. This inequality can be expanded as

$$ab(A_kC_k - B_ka^kk) + a^k(-c^kk + aC_k) + b(-B_ka^k + A_kc^k) < 0.$$

Given that $A_kC_k < B_ka^{k-1}k$, it is enough to show that $a^k(c^kk - aC_k) + b(B_ka^k - A_kc^k) > 0$. Specifically,

$$\begin{aligned}
&a^k(c^kk - aC_k) + b(B_ka^k - A_kc^k) \\
=&\sum_{i=0}^{k-1} a^kc^i(c^{k-i} - a^{k-i}) + b^{i+1}(a^kc^{k-i-1} - a^{k-i-1}c^k) \\
>&\sum_{i=0}^{k-1} a^kc^i(c^{k-i} - a^{k-i}) + a^{i+1}(a^kc^{k-i-1} - a^{k-i-1}c^k) \\
=&\sum_{i=0}^{k-1} a^k(-c^ia^{k-i} + a^{i+1}c^{k-i-1} = 0.
\end{aligned}$$

■

References

Aghion, P. and J. Tirole. "Formal and Real Authority in Organizations," Journal of Political Economy 105(1), 1997, 1-29.

Alonso, R. and N. Matouschek. "Optimal Delegation," *Review of Economic Studies* 75(1), 2008, 259-293.

Alonso, R., W. Dessein and N. Matouschek. "When does Coordination Requires Centralization?" *American Economic Review* 98(1), 2008, 145-179.

Ambrus, A. and S. Takahashi. "Multi-sender Cheap Talk with Restrictive State Space," *Theoretical Economics* 3(1), 2008, 1-27.

Austen-Smith, D. "Interested Experts and Policy Advice: Multiple Referrals under Open Rule," *Games and Economic Behavior* 5, 1993, 1-43.

Battaglini, M. "Multiple Referrals and Multidimensional Cheap Talk," *Econometrica* 70(4), 2002, 1379-1401.

Chakraborty, A. and R. Harbaugh. "Comparative Cheap Talk," *Journal of Economic Theory* 132, 2007, 70-94.

Chakraborty, A. and R. Harbaugh. “Persuasion by Cheap Talk,” *American Economic Review* 100 (5), 2010, 2361-82.

Che, Y., W. Dessein, and N. Kartik. “Pandering to Persuade,” *American Economic Review* 103(1), 2013, 47-79.

Chen, Y., N. Kartik and J. Sobel. “Selecting Cheap-Talk Equilibria,” *Econometrica* 76(1), 2008, 117-136.

Crawford, V. and J. Sobel. “Strategic Information Transmission,” *Econometrica* 50(6), 1982, 1431-1451.

Dessein, W. “Authority and Communication in Organizations,” *Review of Economic Studies* 69, 2002, 811-838.

Epstein, D. “Partisan and Bipartisan Signaling in Congress,” *Journal of Law, Economics, and Organization* 14(2), 1998, 183-204.

Galeotti, A., C. Ghiglino and F. Squintani. “Strategic Information Transmission in Networks,” *Journal of Economic Theory*, forthcoming.

Gilligan, T. and K. Krehbiel. “Asymmetric Information and Legislative Rules with a Heterogeneous Committee,” *American Journal of Political Science*, 1989, 459-490.

Hagenbach, J. and F. Koessler. “Strategic Communication Networks,” *Review of Economic Studies*, 2010, 77, 1072-1099.

Hori, K. “Organizing Information Transmission with Multiple Agents,” 2006, working paper.

Jindapon, P. and C. Oyarzun. “Persuasive Communication when the Sender’s Incentives are Uncertain,” *Journal of Economic Behavior & Organization*, 2013, 95, 111-125.

Krishna, V. and J. Morgan. “A Model of Expertise,” *Quarterly Journal of Economics* 116, 2001a, 747-775.

Krishna, V. and J. Morgan. “Asymmetric Information and Legislative Rules,” *American Political Science Review* 95, 2001b, 435-452.

Li, M. “Advice from Multiple Experts: A Comparison of Simultaneous, Sequential, and Hierarchical Communication,” *The B.E. Journal of Theoretical Economics (Topics)*, 2010, 10(1), Artical 18.

Matthews, S., M. Okuno-Fujiwara and A. Postlewaite. “Refining Cheap Talk Equilibria,” *Journal of Economic Theory*, 1991, 247-273.

McGee, A. and H. Yang. “Cheap Talk with Two Senders and Complementary Information,” *Games and Economic Behavior* 79, 2013, 181-191.

Melumad, N. and T. Shibano. “Communication in Settings with No Transfers,” *Rand Journal of Economics* 22(2), 1991, 173-198.

Ottaviani, M. and P. Sorensen. “Information Aggregation in Debate: Who Should Speak First,” *Journal of Public Economics* 81, 2001, 393-421.

Quint, D. and K. Hendricks. “Indicative Bidding in Auction with Costly Entry,” 2013, working paper, University of Wisconsin

Rantakari, H. “Governing Adaptation,” *Review of Economic Studies*, 2008, 75(4), 1257-1285.

Yang, H. “Firm Size, Strategic Communication, and Organization,” 2013, working paper, Ohio State University.